Section 4.1 - Polarization

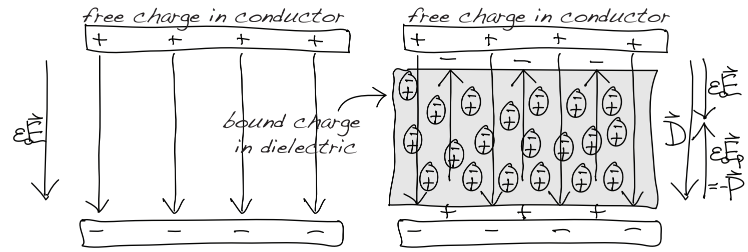
* Overview

~ Ch3: Poisson/Laplace equation more powerful than integrating the field/potential over charge distributions (for example, don't need to know the charge on a conductor)

~ Ch4: Extend formalism to dielectic media (deal with charges in individual atoms)

$$\begin{matrix} \nabla\cdot\vec{E} = \rho/\varepsilon_0 \\ \nabla\times\vec{E} = \vec{0} \end{matrix} \quad \xrightarrow[\varepsilon_0\vec{E}\to\vec{D}]{\varepsilon_0\to\varepsilon} \quad \begin{matrix} \nabla\cdot\vec{D} = \rho_f \\ \nabla\times\vec{E} = 0 \end{matrix}$$

* example: parallel plate capacitor



* Dielectrics

~ charge is bound to neutral atoms

~ not free, but can still polarize

~ either stretching or rotating

* Induced dipoles

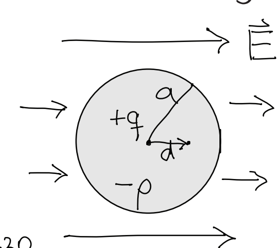
~ field stretches charge apart in atom

~ atomic polarizability tensor

$$\vec{p} = \alpha\vec{E} \qquad \vec{p} = \alpha_\perp \vec{E}_\perp + \alpha_\parallel \vec{E}_\parallel$$

$$\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

* example: nucleus in a cloud of charge



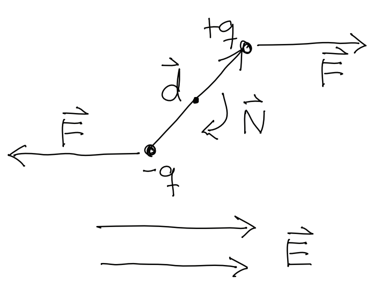
$$E_e = \frac{1}{4\pi\varepsilon_0}\frac{qd}{a^3}$$

$$p = qd = 4\pi\varepsilon_0 a^3 E$$

$$\alpha = 4\pi\varepsilon_0 a^3 = 3\varepsilon_0 v$$

$$\frac{\alpha}{4\pi\varepsilon_0} \approx a^3 \approx 1\,\text{Å}^3 \approx 10^{-30}\,m^3$$

* Dipole in an electric field



$$\vec{N} = \vec{r}_+\times\vec{F}_+ + \vec{r}_-\times\vec{F}_-$$
$$= \vec{d}/2\times q\vec{E} + -\vec{d}/2\times -q\vec{E}$$
$$= q\vec{d}\times\vec{E} = \vec{p}\times\vec{E}$$

$$U = \int N\,d\theta = \int pE\sin\theta$$
$$= -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

$$\vec{F} = \vec{F}_+ + \vec{F}_- = q(\vec{E}_+ - \vec{E}_-)$$
$$= qd\,\frac{\Delta E}{d} = (\vec{p}\cdot\nabla)\vec{E}$$
$$= \nabla(\vec{p}\cdot\vec{E}) \quad \text{if } \vec{p} \text{ const.}$$