Section 4.2 – Polarization Fields

* review: dipole moment, polarization, forces, dielectrics

$\vec{p} = \int dq' \vec{r}' = q\vec{d}$ $\vec{N} = \vec{p}\times\vec{E}$

$d\vec{p} = \vec{P}\,d\tau$ $U = -\vec{p}\cdot\vec{E}$

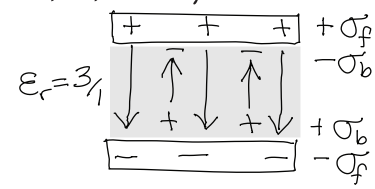
$\vec{F} = q\vec{E}$ $\vec{F} = (\vec{p}\cdot\nabla)\vec{E}$

~ does $(2q)\vec{d} = q(2\vec{d})$?

~ what about $\vec{d}_1 = -\vec{d}_2$?

~ force on a quadrupole?

$\mathcal{E}$, $\mathcal{D}$, P all point down!



$\varepsilon_0 E = D - P = 3 - 2 = 1$

* electric potential from polarization: bound charge

$$V = \frac{1}{4\pi\varepsilon_0}\int_{\mathcal{V}'} \frac{d\vec{p}'\cdot\hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{1}{4\pi\varepsilon_0}\int_{\mathcal{V}'} \vec{P}'d\tau'\cdot\nabla'\frac{1}{\mathscr{r}}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\oint_{\partial\mathcal{V}} \frac{\vec{P}'\cdot\hat{n}'\,da'}{\mathscr{r}} + \int_{\mathcal{V}} \frac{-\nabla'\cdot P'\,d\tau'}{\mathscr{r}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\oint_{\partial\mathcal{V}} \frac{\sigma_b'\,da'}{\mathscr{r}} + \int_{\mathcal{V}} \frac{\rho_b'\,d\tau'}{\mathscr{r}}\right]$$

$$\nabla\frac{1}{r} = \frac{-1}{r^2}\nabla r = \frac{-\hat{r}}{r^2}$$

$$\nabla\frac{1}{\mathscr{r}} = \frac{-1}{\mathscr{r}^2}\nabla\mathscr{r} = \frac{-\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\nabla'\frac{1}{\mathscr{r}} = \frac{d}{d\vec{r}'}\frac{1}{\mathscr{r}} = -\nabla\frac{1}{\mathscr{r}}$$

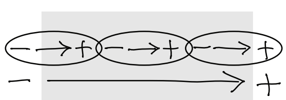
$$\nabla\cdot\frac{\vec{P}}{\mathscr{r}} = \frac{\nabla\cdot\vec{P}}{\mathscr{r}} + \vec{P}\cdot\nabla\frac{1}{\mathscr{r}}$$

bound charge $\boxed{\sigma_b = \vec{P}\cdot\hat{n}}$ $\boxed{\rho_b = -\nabla\cdot\vec{P}}$

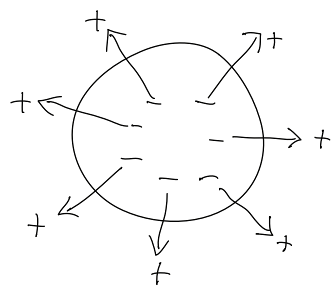
~ uncancelled charge from overlapping dipoles in the polarized (dielectric) medium

* physical interpretation of bound charge

~ polarization forms "dipole chains"



$$\sigma_b = \frac{q_b}{A} = \frac{q\vec{d}\cdot\hat{n}}{Ad} = \vec{P}\cdot\hat{n}$$



~ divergence finds lone charge at end of each chain

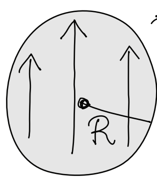
$$\int_{\mathcal{V}} \rho_b\,d\tau = -\oint_{\partial\mathcal{V}} \vec{P}\cdot d\vec{a} = -\int_{\mathcal{V}} \nabla\cdot P\,d\tau \qquad \rho_b = -\nabla\cdot\vec{P}$$

~ what fluxes have we considered so far?

~ what are the similarities and differences between $\vec{E}$ and $\vec{P}$?

~ why are they so similar?

* example 4.2 – bound charge and fields of a sphere with constant polarization $\vec{P}$



$\vec{P} = P\hat{z}$

$\rho_b = -\nabla\cdot\vec{P} = 0$

$\sigma_b = \vec{P}\cdot\hat{n} = P\cos\theta$

$$V_1 = \sum_\ell a_\ell \left(\frac{r}{R}\right)^\ell P_\ell(x)$$

$$V_2 = \sum_\ell a_\ell \left(\frac{R}{r}\right)^{\ell+1} P_\ell(x)$$

$V_1 = V_2$:

$$\sum_\ell a_\ell P_\ell(x) = \sum_\ell a_\ell P_\ell(x)$$

$V_1' - V_2' = \sigma/\varepsilon_0$:

$$\sum_\ell a_\ell\left(\frac{\ell}{R}\left(\frac{r}{R}\right)^{\ell-1} + \frac{\ell+1}{R}\left(\frac{R}{r}\right)^{\ell+2}\right)P_\ell(x) = \frac{P}{\varepsilon_0}\cos\theta$$

$$\sum_\ell a_\ell \frac{2\ell+1}{R} P_\ell(x) = \frac{P}{\varepsilon_0} P_1(x) \qquad a_\ell = \frac{PR}{3\varepsilon_0}\delta_{\ell 1}$$

$$V_1 = \frac{P}{3\varepsilon_0} r\cos\theta = \frac{Pz}{3\varepsilon_0} \qquad \vec{E}_1 = -\nabla V_1 = \frac{-\vec{P}}{3\varepsilon_0}$$

$$V_2 = \frac{PR}{3\varepsilon_0}\frac{R^2}{r^2}\cos\theta = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^3} \quad \text{where } \vec{p} = \tfrac{4}{3}\pi R^3\vec{P}$$

$$\vec{E}_2 = -\nabla V_2 = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}) = \frac{3\vec{p}\cdot\hat{r}\hat{r} - \vec{p}}{4\pi\varepsilon_0 r^3}$$