Section 4.4.3 - Energy in Dielectric Systems

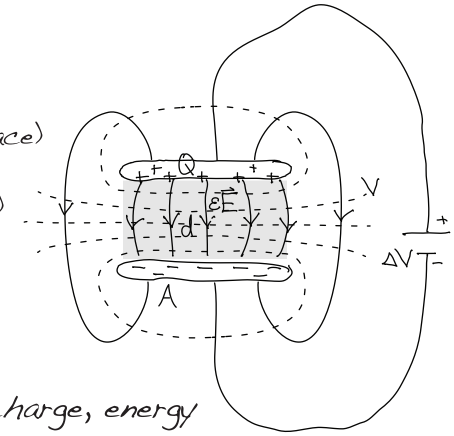
* capacitance = flux/flow

$$C = \frac{Q}{\Delta V} = \frac{\Phi_D}{\mathcal{E}_F} = \frac{\varepsilon \Phi_E}{\mathcal{E}_E} \approx \frac{\varepsilon A}{d}$$

$$\varepsilon_0 \vec{E} \to \vec{D} \qquad \varepsilon_0 \to \varepsilon = \varepsilon_0 \varepsilon_r$$

$Q = \int d\vec{a} \cdot \vec{D} = \Phi_D$ (closed surface)

$\Delta V = \int d\vec{l} \cdot \vec{E} = \mathcal{E}_E$ (open path)



* energy in a capacitor $\quad W = \frac{1}{2} C V^2 = \frac{1}{2} Q V$
  ~ where does the 1/2 come from?
  ~ ε_r (dielectric const) enhancement factor of capacitance, charge, energy

* energy in the electric field = flux × flow $\quad W = \frac{\varepsilon_0}{2} \int E^2 d\tau \to \frac{\varepsilon}{2} \int E^2 d\tau = \frac{1}{2} \int \vec{D} \cdot \vec{E} \, d\tau$
  ~ proof: $V \xrightarrow{d} \vec{E}$ or $\vec{D} \xrightarrow{d} \rho_f$

$$\Delta W = \int \Delta \rho_f V d\tau = \int (\nabla \cdot \Delta \vec{D}) \quad d\tau = \oint d\vec{a} \cdot (\Delta \vec{D} V) - \int \Delta \vec{D} \cdot \nabla V \, d\tau = \int \Delta \vec{D} \cdot \vec{E} d\tau$$

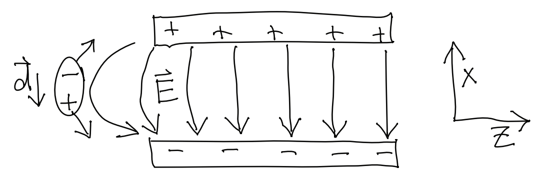
  ~ for a linear dielectric (linear materials),

$$\Delta W = \int \varepsilon \Delta \vec{E} \cdot \vec{E} \, d\tau = \int \frac{\varepsilon}{2} \Delta E^2 d\tau = \Delta \frac{1}{2} \int \vec{D} \cdot \vec{E} \, d\tau$$

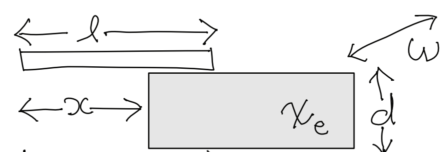
$$\boxed{W = \frac{1}{2} \int \vec{D} \cdot \vec{E}}$$

* forces on dielectrics
  ~ force of a fringe field on a dipole
  ~ due to $\frac{\partial E_z}{\partial x}$ which equals $\frac{\partial E_x}{\partial z}$

$$\vec{F} = -\nabla W = \nabla (\vec{d} \cdot \vec{E}) = \vec{d} \times (\cancel{\nabla \times \vec{E}}) + (\vec{d} \cdot \nabla) \vec{E}$$



* Example: dielectric being pulled into a capacitor



$$C = C_1 + C_2 = \varepsilon_0 \frac{x w}{d} + \varepsilon_0 (1 + \chi_e) \frac{(l - x) w}{d} = \frac{\varepsilon_0 w}{d} (\varepsilon_r l - \chi_e x)$$

$$F = -\nabla W = -\frac{d}{dx} \frac{1}{2} C V^2 = -\frac{d}{dx} \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{C^2} \frac{dC}{dx}$$

$$= \frac{1}{2} V^2 \frac{dC}{dx} = -\frac{\varepsilon_0 w \chi_e}{2d} V^2$$

  ~ with constant V, the force would be the same $\quad F = -\nabla W$
  but in this case W would increase