Conservation of Charge

* symmetries: $\frac{d}{dt}\underbrace{\frac{\partial T}{\partial \vec{v}}}_{\vec{p}} - \underbrace{-\frac{\partial V}{\partial \vec{x}}}_{\vec{F}} = 0$ (Lagrange) $\frac{\partial T}{\partial \vec{v}} = \frac{d}{d\vec{v}}\frac{1}{2}m\vec{v}^2 = m\vec{v} = \vec{p}$

$$\frac{d\vec{p}}{dt} = m\vec{a} = \vec{F} = -\frac{\partial V}{\partial \vec{x}}$$

- if $V(\vec{x})$ is translation invariant (symmetric) then $\vec{p}$ is conserved. (in complete system). N III: $F_{21} = -F_{12}$
- if laws of physics (forces) are time-invariant then E is conserved. (potential energy stored in force)

* Noether's theorem: symmetries $\Leftrightarrow$ conserved currents.

- mass ?

(quantity is conserved; but it can move around).

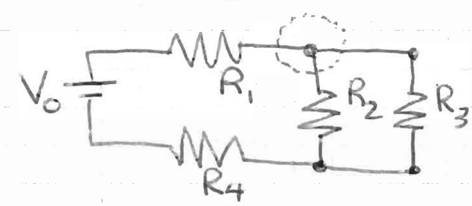
- charge ? what is the symmetry ?

Gauge transformations $\vec{E} = -\nabla V$ the same for different V_0's (ground potential)

* Kirchoff's rules are conservation principles:

a) loop rule: $\sum_{loop} \Delta V_i = -\oint_{\partial A} \vec{E}\cdot d\vec{l} = -\mathcal{E}_E = -\int_A \nabla\times\vec{E}\cdot d\vec{a} = 0$

- conservation of energy.



b) node rule: $\sum_{node} I_i = 0 = \oint_{\partial V} \vec{J}\cdot d\vec{a} = \int_V \nabla\cdot J\, d\tau$

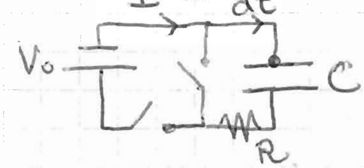
- conservation of charge.

$V_2 = V_3$

$I_1 = I_2 + I_3$

- what about a capacitor?

$I = \frac{dQ}{dt}$



top plate has current in, but no current out.

* current element: $dq = q_i = \lambda dl = \sigma da = \rho d\tau$ (charge).

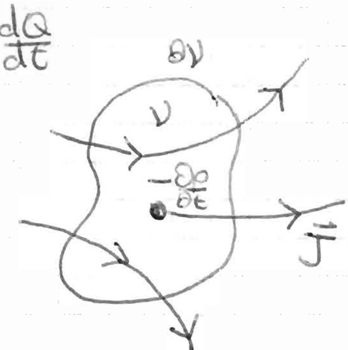
$\vec{v}dq = \vec{v}_i q_i = I d\vec{l} = \vec{K} da = \vec{J} d\tau$ (current).

$\vec{I} = \vec{v}\lambda = \frac{\Delta q}{\Delta t}$ $\vec{K} = \vec{v}\sigma = \frac{\Delta q}{\Delta x \Delta t}$ $\vec{J} = \vec{v}\rho = \frac{\Delta q}{\Delta a \Delta t}$ (current density)

$I = \int \vec{K}\times d\vec{x}$ $I = \int \vec{J}\cdot d\vec{a}$

* continuity equation $I = \oint_{\partial V} \vec{J}\cdot d\vec{a} = \int_V \frac{\partial\rho}{\partial t} d\tau = -\frac{dQ}{dt}$

$$\boxed{\nabla\cdot J = -\frac{\partial\rho}{\partial t}}$$



- 4-vector: $(c\rho, \vec{J}) = J^\mu$

$\frac{1}{c}\frac{\partial}{\partial t}c\rho + \nabla\cdot\vec{J} \equiv \vec{\Box} J = 0$ $(\partial_\mu J^\mu = 0)$

Conductance

symmetries:
continuity → Maxwell eq. → c.
motional $\mathcal{E}$ → Einstein Special Rel.

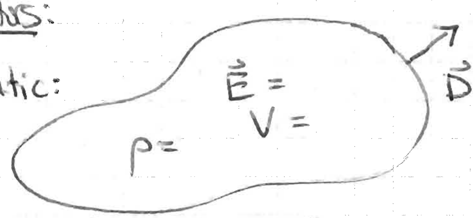
* review: $q \leftrightarrow \lambda \leftrightarrow \sigma \leftrightarrow \rho$

$q\vec{v} \leftrightarrow I \leftrightarrow K \leftrightarrow \vec{J}$

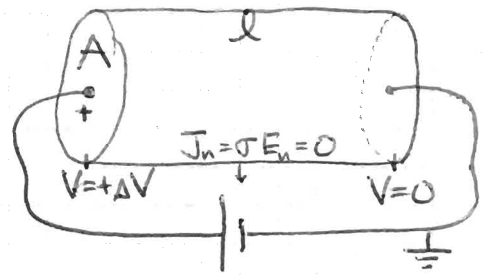
continuity: $\frac{\partial}{\partial t}\rho + \nabla \cdot J = 0$ ⇒ steady currents?

* conductors:

- static:



- steady current: RESISTOR



- what if $\Delta V \neq 0$?

$m\vec{a} = \vec{F} = q\vec{E}$

current $\boxed{\vec{J} = \sigma\vec{E}}$ 2nd constitutive equation

- resistor vs. CRT?

$b\vec{v}_d = -\vec{F}_f = \vec{F}_e = q\vec{E}$ terminal velocity (drift).

$\vec{J} = \rho_f \vec{v}_d = \underbrace{\frac{\rho_f q}{b}}_{\sigma} \vec{E}$

- Drude law: "bumper cars"

$v_d = \frac{\langle \frac{1}{2}at^2 \rangle}{\langle t \rangle} = a\,t = \frac{qE}{m} \cdot \frac{\lambda}{v_{rms}}$

so $b = \frac{m v_{rms}}{\lambda}$ $\boxed{\sigma = \frac{(\rho_f q)}{b} = \frac{(n f q^2)\lambda}{m\, v_{rms}}}$

(classical Drude law.) ⇒ QM?

t = time between collisions.
λ = mean free path.
nf = atomic density × #charge carriers/atom.

$\nabla^2 V = 0$ B.C.'s?

sol'n: $V = \Delta V \cdot z/\ell$.

$I = \vec{J} \cdot \vec{A} = \sigma E A = \sigma A\, \Delta V$

$= \frac{\Delta V}{R}$ $\boxed{R = \frac{\rho \ell}{A} = \frac{\ell}{\sigma A}}$ ρ = resistivity, σ = conductance

$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$

vs. CAPACITOR

$Q = C\,\Delta V$ $\boxed{C = \varepsilon \frac{A}{\ell}}$

$U = \frac{1}{2} Q \Delta V = \frac{1}{2}\frac{Q^2}{\Delta V} = \frac{1}{2} C \Delta V^2$

vs. INDUCTOR

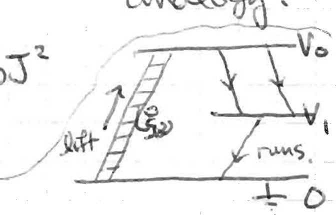
... to be continued...

* power dissipation (power density)

$P = \vec{F} \cdot \vec{v}_d = q\vec{E} \cdot \vec{v}_d$ $\dot{u} = \frac{du}{dt} = \frac{\Delta P}{\Delta \tau} = \rho_f \vec{v}_d \cdot E = \vec{J} \cdot \vec{E} = \sigma E^2 = \rho J^2$

compare: $u = \frac{\Delta W}{\Delta \tau} = \frac{1}{2}\vec{D} \cdot \vec{E} = \frac{1}{2}\varepsilon E^2$

* ski lift analogy:



* relaxation time

$-\frac{\partial \rho_f}{\partial t} = \nabla \cdot \vec{J} = \frac{\sigma}{\varepsilon} \nabla \cdot \vec{D} = \frac{\sigma}{\varepsilon}\rho_f(t)$ ⇒ $\rho = \rho_0 e^{-\sigma/\varepsilon\, t}$ $\tau = \frac{\varepsilon}{\sigma} = RC$

for copper, $\tau = \frac{\varepsilon}{\sigma} = \frac{1/376.7\, c\cdot\Omega}{1/1.678\, \mu\Omega\cdot cm} = 0.445\, Å/c = 1.45 \times 10^{-19}\, s.$