University of Kentucky, Physics 416G
Problem Set #4 (Rev. E) due Wednesday, 2011-10-05

1. In class are discussing five equivalent formulations of electrostatics: (I) $\boldsymbol{E} = \int đq'\,\hat{\boldsymbol{\mathscr{r}}}/4\pi\epsilon_0\,\mathscr{r}^2$; (II) $\Phi_E = Q/\epsilon_0$, $\mathcal{E}_E = 0$; (III) $\nabla\cdot\boldsymbol{E} = \rho/\epsilon_0$, $\nabla\times\boldsymbol{E} = \boldsymbol{0}$; (IV) $V = \int đq'/4\pi\epsilon_0\,\mathscr{r}$; (V) $\nabla^2 V = -\rho/\epsilon_0$. In this exercise we explore the connections between the first three and one new formulation. We will save the last two for the next problem set.

a) Show that (I) $\Rightarrow$ (II) by integration of $\boldsymbol{E}$ over an arbitrary closed surface and path.

b) Show that (I) $\Leftarrow$ (II) by the superposing the fields obtained by applying Gauss' law on point charge elements $đq'$ at each point $\boldsymbol{r}'$ in space.

c) Show that (I) $\Rightarrow$ (III) by taking the divergence and curl of (I).

d) Show that (II) $\Leftrightarrow$ (III).

e) Derive a sixth formulation (VI) by calulating $\nabla^2\boldsymbol{E}$. Hint: take derivatives of the two equations in (III). Thus you are showing that (III) $\Rightarrow$ (VI). This completes the symmetry of the differential vs. integral formulations so that (I) $\Leftrightarrow$ (VI) corresponds to (II) $\Leftrightarrow$ (III) and (IV) $\Leftrightarrow$ (V).

f) Identify the triangle of six formulas that allow direct calculation of each of the quantities $\{\rho,\,\boldsymbol{E},\,V\}$ from either of the other two.

2. Show that the two fields $\boldsymbol{E}_1 = re^{-r^2}\hat{\boldsymbol{r}}$ and $\boldsymbol{E}_2 = e^{-r^2}\left[\hat{\boldsymbol{x}}(x-y) + \hat{\boldsymbol{y}}(x+y) + \hat{\boldsymbol{z}}z\right]$ both have the same divergence $\rho/\epsilon_0 = \nabla\cdot\boldsymbol{E}_1 = \nabla\cdot\boldsymbol{E}_2$. Using the curl, show which field is the actual electric field of the charge distribution ρ.

3. Plot the field lines in the xy-plane of a charge $+Q$ at $(a,0,0)$ and a charge $-2Q$ at $(-a,0,0)$. How many points in space are there where $\boldsymbol{E} = \boldsymbol{0}$? Calcuate their positions (if any)?

4. A flat sheet of copper carries a constant surface charge density of σ on each face. Use Gauss' law to show that the electric field intensity is σ/ϵ_0 just outside the sheet and zero inside.

5. a) Calculate the electric field $\boldsymbol{E}(r)$ for a uniform distribution of charge Q over a spherical shell of radius r' using Coulomb's law. Note the field point r can either be inside or outside the shell.

b) Calculate the same result using Gauss' law and compare with a).

c) For a spherically symmetric charge distribution $\rho(r) = ae^{-r/r_0}$, calculate the normalization constant a so that the total charge is q in all of space.

d) Using part a), calculate $\boldsymbol{E}(\boldsymbol{r})$ for the distribution in c) and compare with Gauss' law.

6. Use Gauss' law to calculate the field of two line charges parallel to the z-axis: $+\lambda$ at $(x=a, y=0)$ and $-\lambda$ at $(x=-a, y=0)$.

Also, Griffiths chapter 2, problems 4, 5, 9, 10, 18.