University of Kentucky, Physics 416G
Problem Set #5 (Rev. C), due Wednesday, 2011-10-12

1. (continuation of HW 4 #1). Now consider the electric potential $V(\boldsymbol{r})$ defined by $\boldsymbol{E} = -\nabla V$.

a) Show that (III) implies $V(\boldsymbol{r})$ is well-defined, and that (III) $\Rightarrow$ (V). Note that the two first-order differential equations (III) are equivalent to the single second-order differential equation (V).

b) Show that the above definition is equivalent to $V(\boldsymbol{r}) = -\mathcal{E}_E \equiv -\int_{\boldsymbol{r_0}}^{\boldsymbol{r}} \boldsymbol{E} \cdot \boldsymbol{dl}$.

c) Using the definition of b), integrate (I) over an arbitrary path in $\boldsymbol{r}$ to show that (I) $\Rightarrow$ (IV).

d) Evaluate the gradient of (IV) to show that (IV) $\Rightarrow$ (I). Thus $\hat{\boldsymbol{\imath}}/\imath^2$ is a perfect differential.

e) Continue calculating the Laplacian of (IV), both directly and by taking the divergence of (I), to show that (IV) $\Rightarrow$ (V) and (I) $\Rightarrow$ (V).

f) Show that (IV) $\Leftarrow$ (V) by the decomposition $\rho(\boldsymbol{r})$ into an integral over $đq'$ of delta functions and finding $\nabla^{-2}\delta^3(\vec{\imath})$ (a Green's function).

g) Rework the proof of the Helmholtz theorem and use it to show that (III) $\Rightarrow$ (I). Note that you need to use both equations of (III) in the Helmholtz theorem to obtain (I). The calculation pieces together parts a), f), and d), involving the tortuous path (III) $\Rightarrow$ (V) $\Rightarrow$ (IV) $\Rightarrow$ (I).

h) Bonus: repeat above for the new formulation (VI): show the four steps of (I) $\Leftrightarrow$ (VI) $\Leftrightarrow$ (V).

2. Sketch the field lines and potentials for the system of three point charges: charge $+q$ at $(0, 0)$, charge $-q$ at $(1, 0)$, and charge $-q$ at $(0, 1)$, all in the xy-plane.

3. The Spallation Neutron Source (SNS) at Oak Ridge National Laboratory accelerates protons to an energy of 1 GeV, to spall (smash) neutrons out of a mercury target, yielding on average 30 free neutrons per incident proton. At the nominal beam current is 1.4 mA, how many neutrons are 'produced' per second? How much power is dumped into the target?

4. a) Show that the potential due to two parallel line charges of opposite polarity is $V = \frac{\lambda}{2\pi\epsilon_0} \ln(\frac{s_1}{s_2})$ where s_1 is the distance from the field point to the negative line, and s_2 is the corresponding distance to the positive line. If your answer is 0/0, try another method.

b) Determine the shape and equation of the equipotential surfaces, assuming that the negative line charge is along the z-axis at $(x, y) = (-a, 0)$ and the positive line charge is at $(+a, 0)$.

5. A spherical charge distribution inside a sphere of radius r_0 is $\rho(r) = \rho_0(1 - r^2/r_0^2)$.

a) Calculate the total charge Q.

b) Calculate the electric field $\vec{\boldsymbol{E}}$ both inside and outside the charge distribution

c) Calculate the electric potential V both inside and outside the charge distribution.

d) At what radius is the magnitude of electric field the greatest?

e) How does the the potential behave at that radius?

6. Calculate the potential energy of a cube of length a on each side with a point charge q at each of the eight vertices.

Also, Griffiths chapter 2, problems 16, 20, 21, 26, 32, 34, 39, 46, bonus: 48, 49, 51.