University of Kentucky, Physics 416G
Problem Set #10 (Rev. C), due Wednesday, 2011-12-07

1. An infinite **cylindrical rod** of radius R centered in the xy-plane and extending along the z-axis has a uniform polarization $\boldsymbol{P} = P\hat{\boldsymbol{x}}$.

a) Calculate the bound surface charge density σ_b.

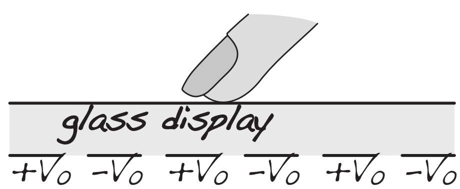
b) Calculate $\boldsymbol{E}_b$ (due to the bound charge) inside and outside the cylinder as a function of $\boldsymbol{P}$.

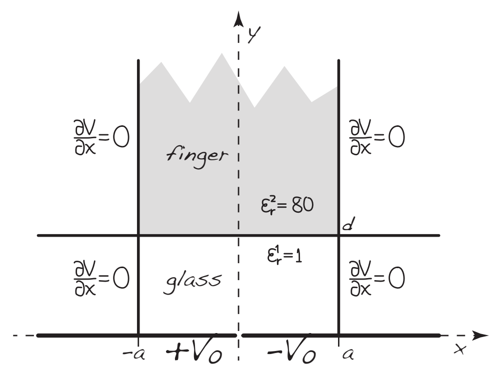
c) Suppose $\boldsymbol{P} = \chi_e\epsilon_0\boldsymbol{E}$, where $\boldsymbol{E}$ is the total electric field inside the rod, placed in an external electric field $\boldsymbol{E}_0 = E_0\hat{\boldsymbol{x}}$. Calculate the values of $\boldsymbol{E}$, $\boldsymbol{P}$, and $\boldsymbol{D}$ inside and outside of the rod.

d) Show that $\epsilon_0 E_{2n} - \epsilon_0 E_{1n} = P_n$ and that $D_{2n} - D_{1n} = 0$ for your solution.

e) Solve the same problem (part c) as a boundary value problem with dielectric constants and $\sigma_f = 0$, and show that the two answers are consistent.

2. In a **capacitive touch-screen**, your dielectric finger modifies the capacitance between two strips of the transparent conductor indium tin oxide (ITO). We will model a single cell (from the middle of one strip to the middle of the next) of this display as an semi-infinite rectangle extending upward from the strips. Use the boundary conditions $\partial V/\partial x = 0$ on the sides at $x = \pm a$ to ensure the correct periodic symmetry.





a) Solve the boundary value problem for the potential without a finger ($\epsilon_r^1 = \epsilon_r^2 = 1$). Note that in this case there is no discontinuity at the glass and the problem can be solved with a single region extending from $0 < y < \infty$. Calculate the first two nonzero terms of the Fourier expansion.

b) Solve for the potential when a finger is present. Assume the glass thickness is d, and your finger of pure water ($\epsilon_r^2 \approx 80$) fills the whole region beyond the glass.

c) Plot the equipotentials and field lines in the region with and without your finger.

d) Calculate the capacitance/length with and without your finger, as a function of d/a. Don't forget fields on both the top and bottom of the strip. What glass thickness do you recommend?

e) If the voltage on each wire is $\pm V = \pm 2V$, what difference in charge must the device be able to measure? ($d = a = 3$ mm, 1 cm long) If the strips were joined into an LRC circuit, what frequency shift $\Delta\omega$ would the device need to discriminate?

Also Griffiths chapter 4, problems 16, 18, 20, 21, 22, 24, 28, 29, 30, 32, 33, and 34.

3. [bonus] a) Calculate the electric potential due to a point multipole $Q^{(\ell)}_{\text{int}}$ (see HW 8 #1) placed at the origin, embedded in a material of dielectric constant ϵ_r. Hint: put the multipole inside a spherical bubble of radius δ, solve for the external potential, and take the limit as $\delta \to 0$. This is similar to the proof of the Clausius-Mossotti relation in HW 9 #2. Note: the solution to the problem depends on the shape of the for the bubble the multipole is placed in, even in the limit as $\delta \to 0$.

b) Now let the point point multipole $Q^{(\ell)}_{\text{int}}$ be embedded at the center of a sphere of radius R and dielectric constant ϵ_r. Calculate the electric potential inside and outside the sphere. Hint: use the result from part a) to form the boundary condition as $r \to 0$.

4. [bonus] Thermal conductivity K defined by $\boldsymbol{q} = -K\nabla T$, is similar to electrical conductivity σ, but for the flow of heat $dQ/dt = \boldsymbol{q}\cdot\boldsymbol{da}$ across a thermal gradient $-\nabla T$, instead of charge $I = dq/dt = \boldsymbol{J} \cdot \boldsymbol{da}$ across and potential gradient $\boldsymbol{E} = -\nabla V$ in the electrical conduction equation $\boldsymbol{J} = -\sigma\nabla V$. Note the difference between the symbols: Q (heat energy), $\boldsymbol{q}$ (heat flux), and q (charge).

a) Show that $K = \frac{1}{2}kNv_{\text{rms}}\lambda$, assuming classical diffusion, where k is Boltzman's constant, N is the electron density, and λ the mean free path of electrons.

b) Show that the relation between electrical and thermal conductivity is $K/\sigma T = 3k^2/2e^2$.

5. [bonus] a) Show that the power density in a resistor is $dP/dl = I^2\rho/A$, where ρ is the resistivity and A the cross sectional area of the resistor.

b) Show the similar result for a surface resistor: $dP/da = K^2\rho/d$ where d is the thickness of the surface resistor, and $dP/d\tau = J^2\rho$ for bulk resistance.