

University of Kentucky, Physics 420
Homework #2, Rev. A, due Monday, 2015-09-21

1. Calculate and plot the **Fourier transform** $\psi(x) = \int_{-\infty}^{\infty} dk A(k)e^{ikx}$ of a square frequency distribution $A(k) = A_0$ if $k_1 < k < k_2$ and 0 otherwise. Calculate the uncertainty Δk in frequency. Show that the Heisenberg uncertainty principle holds for this wavefunction, but very badly!

2. Surface waves—According to the [Airy theory](#) of gravity waves, the dispersion relation for water waves is $\omega^2 = gk \tanh(kh)$, where h is the water depth and g is acceleration due to gravity. Calculate the phase velocity $v_\phi = \omega/k$, group velocity $v_g = d\omega/dk$, and rate of dispersion $\beta = d^2\omega/dk^2$ for water wave packet centered about frequency k_0 in the shallow water ($kh \ll 1$) and deep water ($kh \gg 1$) limits. Do individual crests move forward or backward within the wave packet? What if anything is the significance of negative dispersion β ? Calculate the same coefficients at short wavelengths, where surface tension displaces gravity as the restoring force. Now the dispersion relation becomes $\omega = \sqrt{k^3\gamma/\rho}$, where $\gamma \approx 73 \text{ mN/m}$ and $\rho \approx 1 \text{ g/cm}^3$ are the surface tension and density of water, respectively. Do surface ripples go forward or backward in their wave packet?