University of Kentucky, Physics 520
Homework #6, Rev. A, due Monday, 2015-10-26

1. A **bouncy neutron** is trapped in the vertical z-direction on a perfectly reflecting horizontal neutron mirror ($V = \infty$ for $z < 0$) and by the earth's gravitational potential $V = mgz$ where m is the mass of the neutron and g is the acceleration due to gravity. Ignore the independent uniform horizontal motion in the x- and y-directions (see Nature **415** 299 (2002)).

a) Write down the Hamiltonian for this system and solve for the energy eigenstates. *Hint:* Substitute the dimensionless parameter $\zeta = z/z_0 - \zeta_n$ into the TISE to obtain the Airy equation, $d^2\psi/d\zeta^2 - \zeta\psi = 0$, with Airy function solutions $Ai(\zeta)$ and $Bi(\zeta)$. Quantize the energy by applying boundary conditions to show that ζ_n is the n^{th} zero of $Ai(\zeta)$.

b) Calculate the quantum gravitational height scale z_0 [μm]. Calculate the total energy E_n [peV], frequency $\omega_n/2\pi$ [Hz], and the classical turning points z_n [μm] for the three lowest quantum states $n = 1, 2, 3$. Plot the energies E_n and wavefunctions $\psi_n(z)$ on the graph of $V(z)$ as usual.

c) Given the initial wave function $\psi_0(z) = 1/\sqrt{z_0}$ if $0 < z < z_0$ and 0 elsewhere, calculate the initial amplitudes of the first three energy states at $t = 0$, and at any later time t. Using these three states, calculate the expectation value of energy $\langle E \rangle$, and compare with the expectation value. What frequency should one vibrate the mirror in order to excite a neutron from the ground state to the first excited state?

2. Show that if operators A and B are linear, then $A + B$, AB, and $[A, B]$ are linear. Which of the following operators are linear? **a)** $A_1\psi(x) = x^3\psi(x)$, **b)** $A_2\psi(x) = x(d/dx)\psi(x)$, **c)** $A_3\psi(x) = \lambda\psi^*(x)$, **d)** $A_4\psi(x) = e^{\psi(x)}$, **e)** $A_5\psi(x) = d\psi/dx + a$, **f)** $A_6\psi(x) = \int_{-\infty}^{x} dx'\psi(x')x'$.

3. Verify using the matrices for x and p in the hamonic oscillator energy eigenbasis $|n\rangle$, that $[x, p] = i\hbar$. Construct the matrix for $H = p^2/2m + \frac{1}{2}mw^2x^2$ from the matrices for x and p. Use matrices to calculate the expectation value of x^4 in the ground state: $\langle 0|x^4|0\rangle$.