L03-Blackbody Radiation: Probability Distributions

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* Blackbody history (Gasiorowicz 1-1, wiki: Planck's Law)

1858 Balfour Stewart - Lamp black absorbs most
1859 Kirchoff - universality of emissivity / absorption
- introduced Blackbody radiation - noted importance
1865 Tyndall - observed peak
1880, 81-6 Crova, Langley - better data 3-d data
1894 Wien - $f(\lambda T)/\lambda^5$ based on thermal laws
1896 Wien - $e^{-c/\lambda T}/\lambda^5$ high freq. cutoff
1898 Lummer & Kurlbaum high quality data
to distinguish exact nature of Planck's law
1899 Lummer & Pringshein : $\sim \lambda^{-5} e^{-c/\lambda T}$
1900 Rayleigh heuristic formula: $c_1 T \lambda^{-4} e^{-c_2/\lambda T}$
1900 Planck : $2hc^2/\lambda^5 \left(e^{hc/\lambda kT}-1\right)^{-1}$ emperical $C\lambda^{-5}/e^{c/\lambda T}-1$
combined Rayleigh + Wien using entropy
1905 Rayleigh-Jeans law: $\frac{8\pi}{\lambda^4} kT = \frac{8\pi \nu^2}{c^3} kT$
1911 Eherenfest - "UV catastrophe" Wien-Nobel prize
1918 Planck accepts physical quantization - Nobel prize.

* switching variables in a distribution

$$u_\nu(\nu)\,d\nu = u_\lambda(\lambda)\,d\lambda$$

$$c = \nu\lambda$$

$$u_\lambda = f_\nu(\nu)\left|\frac{d\nu}{d\lambda}\right| \qquad \frac{d\nu}{d\lambda} = \frac{-c}{\lambda^2}$$

$$= u_\nu\left(\frac{c}{\lambda}\right)\cdot\frac{+c}{\lambda^2}$$ (-) switch direction

* energy density to flux:
(same as vacuum conductivity of an aperture)
imagine particles in 4π, what is flux through hole ?

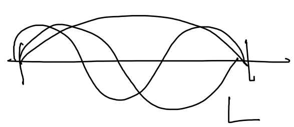
$$d\Phi = \vec{J}\cdot d\vec{a} = \rho\vec{v}\cdot d\vec{a} = \rho\int\frac{v\,d\Omega}{4\pi}\hat{r}(\theta,\phi)\cdot d\vec{a}$$

velocity dist. $\quad$ $\cos\theta$

$$= \frac{\rho v}{4\pi}\int d\phi \sin\theta\, d\theta \cdot \cos\theta = \frac{\rho v}{4\pi}\cdot 2\pi \int_0^1 u\, du = \frac{1}{4}\rho v$$

1/2 particles to the right, 1/2 from $\cos\theta$

* radiative degrees of freedom – modes in a box
quantization of wavelength due to boundary cond.



$$\frac{\lambda}{2} = \frac{L}{n} \qquad \nu = \frac{c}{\lambda} = \frac{cn}{2L} \qquad n = \frac{2L}{c}\nu$$

$$\#\text{ of modes} = \sum_{n_x n_y n_z} \cong \int d^3n = \int n^2 dn\, d\Omega = 4\pi\left(\frac{2L}{c}\right)^3 \int \nu^2 d\nu$$

density of modes $\quad g(\nu)\,d\nu = \frac{32\pi}{c^3}\nu^2 d\nu \cdot \frac{2}{8}$ $\frac{\text{polarization states}}{\text{octants}}$

* Rayleigh-Jeans law: – equipartition

$$u(\nu) = g(\nu)\bar{\varepsilon}(\nu) \qquad \frac{\text{energy}}{\text{volume}} = \frac{\text{modes}}{\text{volume}} \cdot \frac{\text{energy}}{\text{mode}}$$

$$\bar{\varepsilon}(\nu) = \frac{\int \varepsilon f_B(\varepsilon)}{Z \equiv \int f_B(\varepsilon)}$$ weighted average
$\leftarrow$ normalization (Partition fn)

$f_B = e^{-E/kT}$ Boltzman distribution (statistical mech).

$$Z\left(\beta = \frac{1}{kT}\right) = \int_0^\infty e^{-\beta\varepsilon} d\varepsilon = \frac{-1}{\beta} e^{-\beta\varepsilon}\Big|_0^\infty = \frac{1}{\beta}$$

$$\int \varepsilon e^{-\varepsilon/kT} = \int -\frac{\partial}{\partial\beta} e^{-\beta\varepsilon} = -\frac{\partial}{\partial\beta} Z(\beta) = \frac{1}{\beta^2}$$

$$\langle\varepsilon\rangle = -\partial_\beta \ln Z = \frac{-\partial Z/\partial\beta}{Z(\beta)} = \frac{1/\beta^2}{1/\beta} = 1/\beta = kT$$

equipartion theorem: $\frac{1}{2}kT$ per D.O.F. (what are they?)

thus $\quad u(\nu) = \frac{8\pi\nu^2}{c^3} kT$

* Planck's model for his emperical fit:
  assume that energy can only be emitted in quanta: $\varepsilon = h\nu$ $\quad h = 6.63\times10^{-34}\,\mathrm{J\cdot s}$

$$\bar{\varepsilon} = \frac{\sum_{n=0}^{\infty} \varepsilon_n f_B}{Z = \sum_{n=0}^{\infty} f_B} \qquad \varepsilon_n = nh\nu$$

$$Z = \sum_{n=0}^{\infty} e^{-\beta\varepsilon_n} = \sum_{n=0}^{\infty}\left(e^{-\frac{h\nu}{kT}}\right)^n = \frac{1}{1-e^{-\frac{h\nu}{kT}}} \qquad \text{let } x = \frac{h\nu}{kT}$$

$$-\frac{\partial Z}{\partial\beta} = -\frac{\partial}{\partial\beta}(1-e^{-\beta h\nu})^{-1} = (1-e^{-\beta h\nu})^{-2}\cdot e^{-\beta h\nu}\cdot -h\nu$$

$$\langle\varepsilon\rangle = \frac{-\partial_\beta Z}{Z} = \frac{(1-e^{-\beta h\nu})^{-2}\cdot e^{-\beta h\nu}\cdot -h\nu}{(1-e^{-\beta h\nu})^{-1}} = \frac{h\nu}{e^{h\nu/kT}-1}$$

$$\xrightarrow{h\nu\to 0} kT \quad \text{classical limit}$$

Planck's law: $$u(\nu) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT}-1} = \frac{8\pi h\nu^3}{c^3(e^{h\nu/kT}-1)}$$

$$u(\lambda) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$$

* Wien's law vs. Wien, Planck, Rayleigh-Jeans:

$$u(\lambda) = \lambda^{-5} f(kT) \qquad \text{[general thermodynamics]}$$

Wien: $f(kT) = 8\pi hc\, e^{-\frac{hc}{\lambda kT}}$ good as $\lambda\to 0$

Rayleigh-Jeans: $f(kT) = 8\pi\lambda kT$ $\quad \lambda\to\infty$

Planck: $f(kT) = \dfrac{8\pi hc}{e^{hc/\lambda kT}-1}$ every where (above approximations)

* Wien's law $\to$ Wien's displacement law

$$u(\lambda,T) = \lambda^{-5}f(x) = T^5\frac{f(x)}{x^5} \qquad x = \lambda T$$

$$\left.\frac{du}{d\lambda}\right|_T = T^5\frac{d}{dx}\frac{f(x)}{x^5}\cdot\frac{dx}{d\lambda} = 0 \quad \text{when } \frac{d}{dx}\frac{f(x)}{x^5} = 0$$

ie $\lambda T = x_m$ independent of T or λ separately

x_m: $\frac{\partial}{\partial x} \frac{8\pi hc}{e^{hc/kx}-1} / x^5 = 0$ $\frac{d}{dy} \frac{y^5}{e^y - 1} = 0$ @ $y_{max} = \frac{hc}{k\lambda_{max} T}$

* Stefan-Boltzman law:

$$U(T) = \int u_\nu(T,\nu)\, d\nu = \int \frac{8\pi h \nu^3\, d\nu}{c^3(e^{h\nu/kT}-1)} = \frac{8\pi h}{c^3}\left(\frac{kT}{h}\right)^4 \underbrace{\int_0^\infty \frac{x^3 dx}{e^{x}-1}}_{\pi^4/15}$$

or $u_\lambda(T,\lambda)\, d\lambda$?

$$= \sigma_{SB} T^4 \qquad \sigma_{SB} = \frac{8}{15} \frac{\pi^5 k^4}{(hc)^3} = 7.56 \times 10^{-16}\ J/K^4 m^3$$