L05-Bohr Model: Correspondence Principle

Friday, September 4, 2015 10:07 AM

* classical Rutherford model:

- atomic spectra
- J.J. Thompson "plumb puding" model
  modes of vibration
- "radiation catastrophe"

* problems with Bohr model

- only: H + H-like atoms, K, L series X-rays.
- wrong angular momentum
- ad-hoc quantization
- no prediction of spectral intensity.
- no description of radiative process

* Fundamental constants

$kT = 25$ meV [300 K]

$e^2/4\pi\epsilon_0 = 1.44$ eV·nm "electric" energy

$\hbar c = 197$ eV·nm [GeV·fm] "quantum" energy

$\alpha = e^2/4\pi\epsilon_0 \hbar c = 1/137$ — ratio —

$m_e c^2 = 0.511$ MeV mass energy

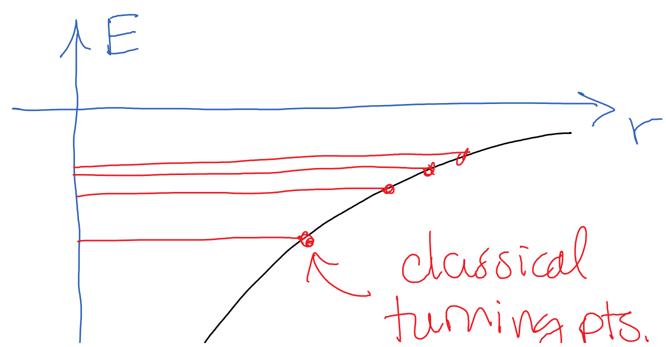
$E_0 = \frac{1}{2} m_e c^2 \cdot \left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)^2 = 13.6$ eV ionization energy.

* Virial Theorem $\langle T \rangle$ vs $\langle V \rangle$

$V = kr^n$

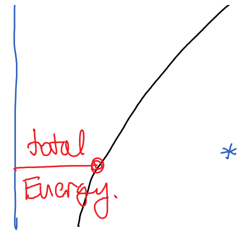
$\vec{F} = -\nabla V = -knr^{n-1}\,\hat{r}$

$= -\frac{mv^2}{r}\,\hat{r}$ centripetal



$$= -\frac{mv^2}{r}\hat{r} \quad \text{centripetal}$$

$$n\langle V\rangle = 2\langle T\rangle$$



classical turning pts.

* what about kinetic energy at radial turning point?

Hydrogen: $n=-1$ $\quad \langle T\rangle = -\frac{1}{2}V \qquad E = \langle T\rangle + \langle V\rangle = \frac{1}{2}V$

$$V = -\frac{Ze^2}{4\pi\varepsilon_0 r} \qquad T = \frac{1}{2}mv^2 = -\frac{1}{2}V \qquad v^2 = \frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{mr}$$

* Rydberg formula: $\quad \frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{\nu}{c}$

* Bohr's Postulates

a) "stationary orbits" stable orbits energy E_n
b) quantum transitions – "photon" $E = h\nu = E_{n_i} - E_{n_f}$
c) quantum condition – $L = n\hbar$

$$v_n = \frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{\underbrace{mvr}_{L=n\hbar}} = \frac{Z}{n}\frac{e^2}{4\pi\varepsilon_0\hbar c}\cdot c = \frac{Z\alpha c}{n} \qquad Z < 137!$$

$$r_n = \frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{mv^2} = \frac{n^2}{Z}\underbrace{\frac{\hbar c}{m_e c^2}}_{\lambda_c}\cdot\underbrace{\frac{4\pi\varepsilon_0\hbar c}{e^2}}_{1/\alpha} = \frac{n^2}{Z}a_0$$

$$E_n = -\frac{Ze^2}{2\cdot 4\pi\varepsilon_0 r_n} = -\frac{Z^2}{n^2}\frac{m_e c^2}{2}\cdot\underbrace{\left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)^2}_{\alpha^2} = -\frac{Z^2}{n^2}E_0$$

* correspondence principle:

– frequency of orbital $\cong$ frequency of emission as $n \to \infty$ (classical)

- Ehrenfest' theorem (expectation values classical)

$$\frac{d\langle x\rangle}{dt} = \left\langle \frac{\partial \mathcal{H}}{\partial p} \right\rangle = \frac{\langle p\rangle}{m} \qquad \frac{d\langle p\rangle}{dt} = -\left\langle \frac{\partial \mathcal{H}}{\partial x} \right\rangle = -\left\langle \frac{\partial V}{\partial x} \right\rangle$$