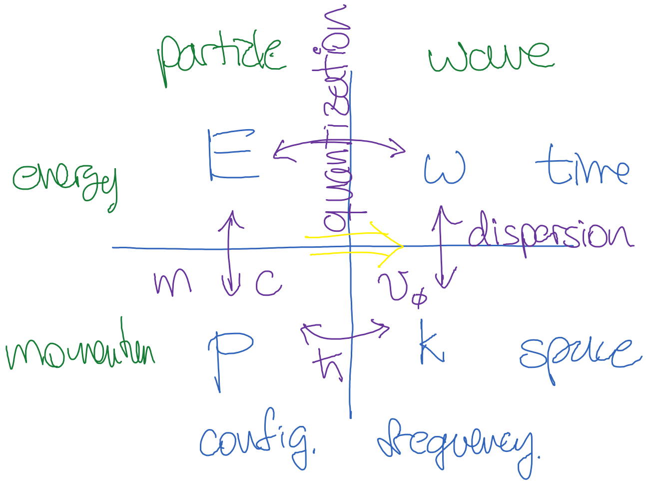
L06-de Broglie waves: Interference

Saturday, September 5, 2015 1:33 PM

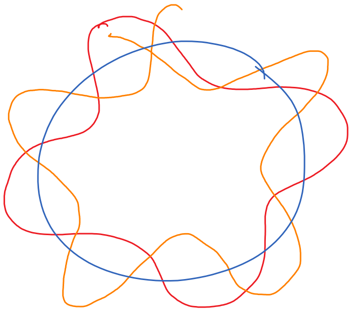
* deBroglie:

$$E = h\nu$$
$$p = h/\lambda$$



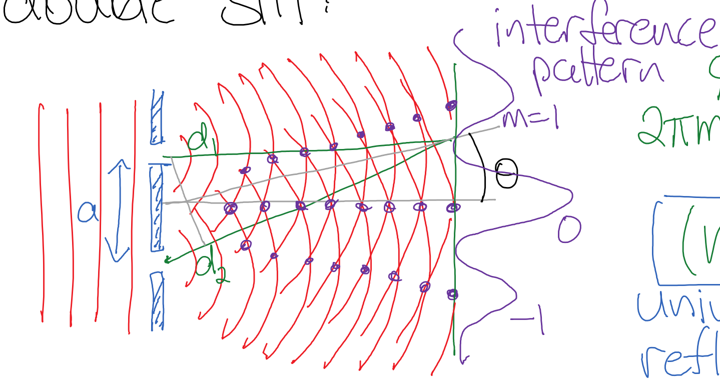
* Bohr model: wave wraps around orbital.

$$2\pi r = n\lambda = nh/p$$
$$\vec{L} = \vec{r}\times\vec{p} = n\hbar\,\hat{z}$$



3-d?

* double slit:



$$\phi = k(d_2 - d_1)$$
$$2\pi m = \frac{2\pi}{\lambda}\, a\cdot\sin(\theta)$$
$$kd = \frac{\omega}{c}\underbrace{nd}_{\text{optical path}}$$

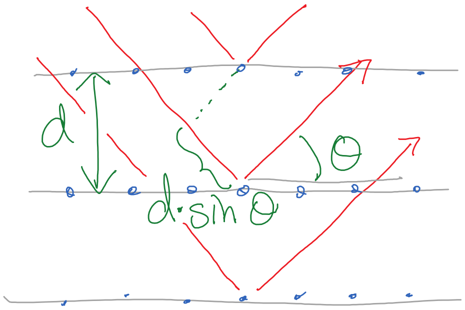
$$\boxed{(n_2\sin\theta_2 - n_1\sin\theta_1)\,d = m\lambda}$$

Universal law for: reflection, refraction, diffraction

- Henry Rowland - diffraction grating
  Rowland circle for focusing lines on film

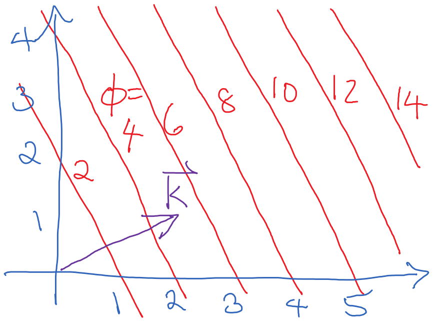
- vs Bragg formula: same!
  reflection from different planes

$$m\lambda = 2d\sin\theta$$



✓ – Dirac-Kapitza effect:
shows true duality between particle-wave.

* plane wave $\Psi(x,t) = \underbrace{\tilde{A}}_{\text{not shown}} e^{i\underbrace{(kx-\omega t)}_{\phi}}$



– prove that $\vec{k} \perp$ wavefront

– what is the "velocity"?

$v_\phi = \frac{\omega}{k}$? $v_x = \frac{\omega}{k_x}$ $v_n = \frac{\omega}{\vec{k}\cdot\hat{n}}$

* why complex amplitudes?

- Schrödinger equation 1st order in time
  - positive probability density
- factorization of time & spatial parts

$\sin(kx-\omega t) = \sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t)$

$e^{i(kx-\omega t)} = e^{ikx}\cdot e^{-i\omega t}$

- eigenfunctions! $\boxed{\frac{d}{dx}} e^{\alpha x} = \alpha e^{\alpha x}$

$\boxed{\frac{d^2}{dx^2}}\sin(kx) = -k^2\cdot\sin(kx)$ $\boxed{\frac{d}{dx}} e^{ikx} = ik\, e^{ikx}$

⊛ OPERATORS extract information about the state

- commutators:

The commutators of 2 Hermitian operators is anti-Hermitian $[\underbrace{\partial_x}_{\text{anti Hermitian.}}, x] = 1$

i restores Hermiticity.

⊛ "COMPLEX interference amplitude" is the ESSENCE of Quantum Mechanics

✓ * when is X a particle and when a wave?

- detected as a particle (probability amplitude)

- propagates as a wave: interference afects detection

✓ * what is the medium of the wave?

- E&M: vacuum, but ε, μ relate E H
  energy $= dW = \vec{F} \cdot d\vec{x} = d\vec{D} \cdot \vec{E} \simeq \frac{1}{2} \varepsilon E^2 = \frac{1}{2} Z H^2$
  quantum: amplitude $\propto$ frequency, $W \sim$ # photons.
- quantum analogs: $dP = |\Psi|^2$ single particle
  properties of the medium? mass