catchup

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$*$ Summary of SHO:

- operator algebra: $\mathcal{H} = \hbar\omega(a_+ a_- + \frac{1}{2})$ $[a_-, a_+] = 1$ $[\mathcal{H}, a_\pm] = \pm\hbar\omega a_\pm$

1) $\mathcal{H}|n\rangle = E_n|n\rangle$ $\mathcal{H} a_\pm|n\rangle = a_\pm \mathcal{H}|n\rangle \pm \hbar\omega\, a_\pm|n\rangle = (E_n \pm \hbar\omega)\, a_\pm|n\rangle$

thus $a_\pm|n\rangle \propto |n\pm1\rangle$ and $E_n = \hbar\omega(n+\frac{1}{2})$, ie $a_+a_- = n$

2) $a_-^\dagger = a_+ \Rightarrow$ $n = \langle n|a_+a_-|n\rangle = \langle a_-n|a_-n\rangle \Rightarrow a_-|n\rangle = \sqrt{n}\,|n-1\rangle$

$n+1 = \langle n|a_-a_+|n\rangle = \langle a_+n|a_+n\rangle \Rightarrow a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

we can now write matrices for a_-, a_+ $\hat{x}, \hat{x}^2, \hat{V}$ $\hat{p}, \hat{p}^2, \hat{T}$ $\hat{H}$

- transition: $a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{p} + m\omega x) = \frac{1}{\sqrt{2}}(\mp\partial_\xi + \xi)$ $\xi = \sqrt{\frac{m\omega}{\hbar}}\, x$

3) $a_-|0\rangle = 0$ $(\partial_\xi + \xi)\psi_0 = 0$ $\psi_0 \propto e^{-\frac{1}{2}\xi^2}$ Gaussian!

4) $|n\rangle = \frac{1}{\sqrt{n!}} a_+^n |0\rangle = \frac{1}{\sqrt{2^n n!}}(-\partial_\xi + \xi)e^{-\frac{1}{2}\xi^2} = \frac{1}{\sqrt{2^n n!}} \underbrace{H_n(\xi)}_{\text{Hermite polys}} e^{-\frac{1}{2}\xi^2}$

$H_0 = 1$ $H_1 = 2\xi$ $H_2 = 4\xi^2 - 2$ $H_3 = 8\xi^3 - 12\xi$

- Frobenius method: Taylor series : ODE + B.C.'s.

5) dimensionless $\mathcal{H}$: $\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2x^2\right)\psi = E\psi \Rightarrow \left(-\frac{d^2}{d\xi^2} + \xi^2 - K\right)\psi = 0$

6) asymptotic form: $\psi = h(\xi)e^{-\xi^2/2}$ $h'' - 2\xi h' + (K-1)h = 0$

7) power series: $h(\xi) = \sum_{j=0}^{\infty} a_j \xi^j$ $a_{j+2} = \frac{2j+1-K}{(j+1)(j+2)}$

8) B.C.'s (truncation): $E_n = \frac{1}{2}\hbar\omega K_n = \hbar\omega(n+\frac{1}{2})$ $\quad a_{j+2}^{(n)} = \frac{-2(n-j)}{(j+1)(j+2)}$
quantization

$$\psi_n(\xi) = \left(\frac{m\omega}{\pi\hbar}\right) \frac{1}{\sqrt{2^n n!}} H_n(\xi)\, e^{-\xi^2/2}$$