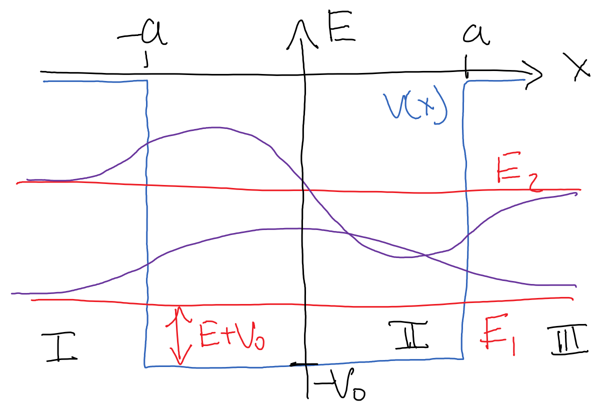
L19 - Finite Square Well

Wednesday, November 4, 2015 08:27



* potential. $V(x) = -V_0\,\Theta(a-|x|)$
- finite: $V=0$ if $|x|>a$
- square: V constant if $|x|<a$

* Bound states:

3 regions need to be sewn together w/ B.C.'s.
- can reduce to one boundary by symmetry: $V(x)=V(-x) \Rightarrow \psi(x)$ either even or odd.

$\psi_{I}(x) = \pm\,\psi_{III}(x)$ obtain by symmetry.
$\psi_{II}(x) = D\cos(lx)$ [even] OR $C\sin(lx)$ [odd]
$\psi_{III}(x) = F e^{-\kappa x}$ [$e^{\kappa x}$ blows up as $x\to\infty$]

$$\frac{\hbar^2 l^2}{2m} = E+V_0 \qquad \frac{\hbar^2\kappa^2}{2m} = E$$

B.C.'s: EVEN OR ODD

i) $\psi_{II}(a) = \psi_{III}(a)$: $D\cos(la) = Fe^{-\kappa a}$ OR $C\sin(la) = Fe^{-\kappa a}$

ii) $\psi'_{II}(a) = \psi'_{III}(a)$: $-lD\sin(la) = -\kappa F e^{-\kappa a}$ OR $lC\cos(la) = -\kappa F e^{-\kappa a}$

solve for: i) $E \to l, \kappa$ and ii) D/F using B.C.'s i, ii
(normalize $\psi(x)$ to get D and F individually).

ratio: $\dfrac{\psi'_{II}(a)}{\psi_{II}(a)} = \dfrac{\psi'_{III}(a)}{\psi_{III}(a)}$ $\quad \tan(la) = \kappa/l$ OR $-\cot(la) = \kappa/l$
$\tan(z) = \dfrac{\sqrt{z_0^2 - z^2}}{z}$ OR $-\cot(z) = \dfrac{\sqrt{z_0^2-z^2}}{z}$

where $z = la$

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Plot[{Tan[z], -Cot[z], Sqrt[(52 / z^2) - 1]},
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where $z \equiv la$

$$\frac{\hbar^2 l^2}{2m} = E + V_0 = \frac{-\hbar^2 \kappa^2}{2m} + V_0$$

$$(\kappa a)^2 = z_0^2 - (la)^2$$

$$z_0^2 = 2mV_0 a^2/\hbar^2$$

$$E_n = \frac{\hbar^2}{2ma^2} z_n^2 - V_0$$

energy states.

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Plot[{Tan[z], -Cot[z], Sqrt[(52 / z^2) - 1]},
 {z, 0, 3 Pi}, PlotRange → {-2, 7}]
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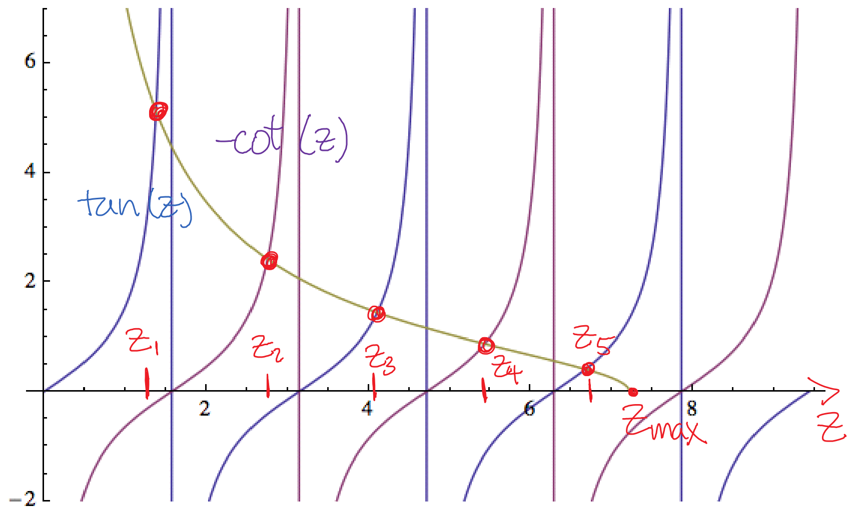


* Scattering States:

$$\psi_I = Ae^{ikx} + Be^{-ikx} \qquad E = \frac{\hbar^2 k^2}{2m}$$

$$\psi_{II} = C\sin(lx) + D\cos(lx) \qquad E + V_0 = \frac{\hbar^2 l^2}{2m}$$

$$\psi_{III} = Fe^{ikx} + Ge^{-ikx}$$

let $G=0$ (incoming from left)

B.C.'s: $x = -a$ $\qquad$ $x = +a$

$$Ae^{-ika} + Be^{ika} = -C\sin(la) + D\cos(la) \qquad C\sin(la) + D\cos(la) = Fe^{ika} + Ge^{-ika}$$

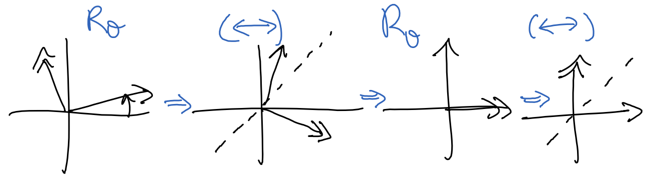
$$ik(Ae^{-ika} - Be^{ika}) = l(C\cos(la) + D\sin(la)) \qquad l(C\underbrace{\cos(la)}_{C_l} - D\underbrace{\sin(la)}_{S_l}) = ik(Fe^{ika} - Ge^{-ika})$$

A = incoming amplitude; B = reflected amp. F = transmitted amp.

$$\begin{pmatrix} l & l \\ ik & -ik \end{pmatrix}\begin{pmatrix} e^{-ika}A \\ e^{ika}B \end{pmatrix} = l\begin{pmatrix} -S_l & C_l \\ C_l & S_l \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} \qquad l\underbrace{\begin{pmatrix} S_l & C_l \\ C_l & -S_l \end{pmatrix}}_{R = R^{-1} = R^T!}\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} l & l \\ ik & -ik \end{pmatrix}\begin{pmatrix} e^{ika}F \\ e^{-ika}G \end{pmatrix}$$

$$\begin{pmatrix} e^{-ika}A \\ e^{ika}B \end{pmatrix} = \underbrace{\begin{pmatrix} l & l \\ ik & -ik \end{pmatrix}^{-1}\begin{pmatrix} C_l^2 - S_l^2 & -2C_lS_l \\ 2C_lS_l & C_l^2 - S_l^2 \end{pmatrix}\begin{pmatrix} l & l \\ ik & -ik \end{pmatrix}}_{\text{Transfer matrix } G}\begin{pmatrix} e^{ika}F \\ e^{-ika}G \end{pmatrix}$$

note: $\left[\overbrace{\underbrace{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}}_{(\leftrightarrow)}\underbrace{\begin{pmatrix}c & -s\\ s & c\end{pmatrix}}_{R_\theta}}^{\text{Transfer matrix } G}\right]^2 =$



$= I$