# L21-Observables: Operators

Wednesday, November 11, 2015 08:34

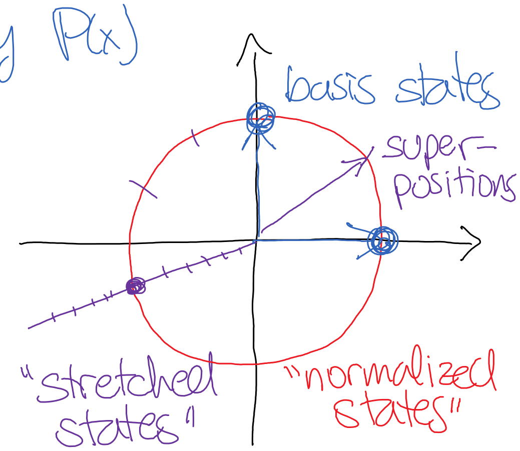
* what does "operate to observe" mean?
  - derives from Born probability amplitude interpretation and the machinery of probability: PDF, expectation value

$\psi(x)$ probability amplitude

$|\psi(x)|^2 = \psi^*\psi$ probability density $P(x)$

$\langle\psi|\psi\rangle = \int dx\, \psi^*\psi = 1$ normalization

"unitary operators evolve (rotate) from one state to another"




* expectation value - weighted average
  note the clean & consistant notation!

$$\langle Q(x)\rangle \equiv \int dx\, P(x)\cdot Q(x) = \int dx\, \psi^*(x)\, Q(x)\, \psi(x) \equiv \langle\psi|Q(x)|\psi\rangle$$

- weighted average of $Q(x)$ probability $|\psi(x)|^2$
- recall doing the same classically for black body radiation!

- the same applies in other coefficients: $\psi(x) = \sum_n c_n \phi_n(x)$
  - $|c_n|^2$ is the probability of being in the state $\phi_n(x)$

let $Q$ have the value $q_n$ for the state $\phi_n(x)$

$$\langle Q\rangle = \sum_n |c_n|^2 q_n = (c_1^* c_2^* \cdots)\begin{pmatrix} q_1 & & \\ & q_2 & \\ & & \ddots \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix} = \langle\psi|\hat{Q}|\psi\rangle$$

again: a weighted average of different measurements:

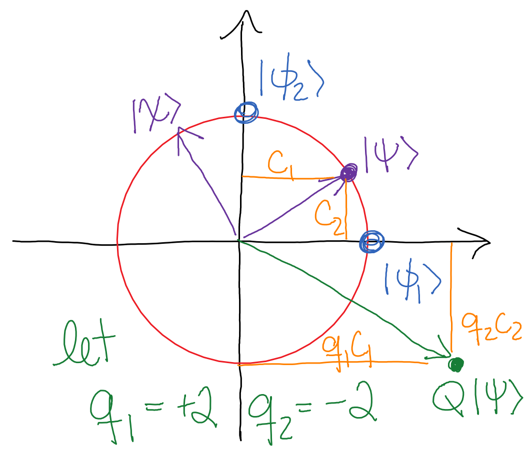
- $\hat{Q}$ is an operator: $|\psi\rangle \to \hat{Q}|\psi\rangle \sim \begin{pmatrix} q_1 & & \\ & q_2 & \\ & & \ddots \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} q_1 c_1 \\ q_2 c_2 \\ \vdots \end{pmatrix}$

$\hat{Q}|\phi_1\rangle = q_1|\phi_1\rangle$ "stretches" $|\phi_1\rangle$ so that $\langle Q\rangle \equiv \langle\phi_1|\hat{Q}|\phi_1\rangle = q_1$

is just the "value of Q" for the state $|\phi_1\rangle$

- reinterpret $\langle Q\rangle = \langle\psi|\hat{Q}|\psi\rangle$
  in terms of stretches:

$$\hat{Q}|\phi_1\rangle = q_1|\phi_1\rangle$$
$$\hat{Q}|\phi_2\rangle = q_2|\phi_2\rangle$$
$$\hat{Q}|\psi\rangle = q_1 c_1|\phi_1\rangle + q_2 c_2|\phi_2\rangle$$
$$\langle\psi|\hat{Q}|\psi\rangle = \left[c_1^*\langle\phi_1| + c_2^*\langle\phi_2|\right]\hat{Q}|\psi\rangle$$
$$= q_1|c_1|^2\cancelto{1}{\langle\phi_1|\phi_1\rangle} + \dots\cancelto{0}{\langle\phi_1|\phi_2\rangle}$$
$$+ q_2|c_2|^2\cancelto{1}{\langle\phi_2|\phi_2\rangle} + \dots\cancel{\langle\phi_2|\phi_1\rangle}$$




- note: $|\phi_1\rangle$ & $|\phi_2\rangle$ are special states for $\hat{Q}$
  they are "eigenstates" or "determinate" states
  - they have a definite associated value $q_i$ of $\hat{Q}$
  - any other state is a superposition of these
  - they are the simplest basis to represent $|\psi\rangle$ for $\hat{Q}$
  - operators do not have to be diagonal, but measurements must be Hermitian.
  - then it is always possible to diagonalize them with a complete orthonormal set of eigenvectors

* Physical measurements: Expansion postulate
$|\psi\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle + \dots$ is a superposition of states with different "values" of $\hat{Q}$
- $|c_n|^2$ is the probability of measuring $q_n$
  thus $\langle Q\rangle = \sum_n |c_n|^2 q_n$
- after measurement, the state will be $|\phi_n\rangle$ depending on which value was obtained.
  now $\hat{Q}|\phi_n\rangle = q_n|\phi_n\rangle$