L22-Expansion postulate & other postulates of Q.M.

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* executive summary of measurement in Q.M.

1) observation is richer than $Q(x,p)$ due to complementarity
   it requires a Hermitian operator $\hat{Q}(x, -i\hbar\frac{\partial}{\partial x})$ on a state $|\Psi(x)\rangle$
2) every observable has special "determinate states" $|\phi_n\rangle \to q_n$
   these are eigenstates: $\hat{Q}\,|\phi_n\rangle = q_n|\phi_n\rangle$
3) any other state is a "superposition" of these (complex linear combo)
   c_n = "probability amplitude"; $|\Psi\rangle$ is a complete set of amplitudes
4) observation is an irreversible projection - collapses the state
5) measurements with same eigenstates (commute) are compatible
6) otherwise "rotation" from one basis to another is unitary
   involving projections (inner products) of each state.
7) Dirac notation treats these opperations in a unified framework

* recall: determinate states & superpositions

$$\langle Q\rangle = \langle\Psi|\hat{Q}|\Psi\rangle = \langle \sum_m c_m\phi_n \,|\, \hat{Q} \,|\, \sum_n c_n\phi_n\rangle$$

$$= \sum_n |c_n|^2 q_n \qquad \text{where } \hat{Q}|\phi_n\rangle = q_n|\phi_n\rangle$$

- compare with original $\langle Q\rangle = \int dx\, |\Psi(x)|^2\, Q(x)$
  what are the "c_n" or "$\Psi(x)$"?
  probability amplitude of "being at" q_n or x

$$\langle Q^2\rangle = \langle\Psi|Q^2|\Psi\rangle = \sum_n |c_n|^2 q_n^2$$

$$\Delta Q = \langle Q^2\rangle - \langle Q\rangle^2 = \sum_n |c_n|^2 q_n^2 - \left(\sum_n |c_n|^2 q_n\right)^2$$
$$= 0 \quad \text{if and only if } c_n = \delta_{nj}$$

- every operator has a basis of determinate states (complete set of orthonormal vectors $|\phi_n\rangle$ with definite q_n

- these are the eigenvectors of $\hat{Q}$: $\hat{Q}|\phi_n\rangle = q_n|\phi_n\rangle$
  energy states: $\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle$ Schrödinger's equation
  position: $\hat{x}|x'\rangle = x'|x\rangle$ Dirac δ: $|x'\rangle \sim \delta(x-x')$
  momentum: $\hat{p}|k\rangle = \hbar k|k\rangle$ Plane waves $|k\rangle \sim e^{ikx}$

- so what are quantum mechanical states $|\psi\rangle$?
  a complete set of probability amplitudes (components)

* Dirac notation:
orthonormality: $\langle \phi_n|\phi_m\rangle = \delta_{nm}$
closure: $\sum_n \underbrace{|\phi_n\rangle\langle\phi_n|}_{P_n\text{, not } |C_n|^2!} = \sum_n P_n = I$

$\hat{Q}|\phi_n\rangle = q_n|\phi_n\rangle$ $\qquad MV = VD$ eigenvalue eq'n

$\langle\phi_m|\hat{Q}|\phi_n\rangle = q_n\delta_{mn}$ $\qquad V^\dagger MV = D$ diagonalization

$\hat{Q} = \sum_n |\phi_n\rangle q_n \langle\phi_n|$ $\qquad M = VDV^\dagger$ spectral representation

Note: orthonormality & closure are eigenvalues & spect. rep of identity!

$$\begin{pmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{pmatrix}\begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix}\begin{pmatrix} - & v_1 & - \\ - & v_2 & - \\ - & v_3 & - \end{pmatrix} = \lambda_1 v_1 v_1^* + \lambda_2 v_2 v_2^* + \dots$$

$$|\psi_m\rangle = \sum_n |\phi_n\rangle\langle\phi_n|\psi_m\rangle = \sum_n |\phi_n\rangle \underbrace{U_{nm}}_{\text{unitary.}}$$