L26-Three Dimensional Space

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$*$ modification from 1-d to 2-d and 3-d space:

- classical: $F=ma \rightarrow \vec{F}=m\vec{a}$ (independent equations)
  more complicated: $\vec{L}=\vec{r}\times\vec{p}$ mixes dimensions

- quantum: $\psi(x) \rightarrow \psi(x,y,z)$, $p=-i\hbar\frac{d}{dx} \rightarrow \vec{p}=-i\hbar\nabla$
  $\hat{H} \rightarrow \frac{-\hbar^2}{2m}\nabla^2+V(\vec{r})$ Laplacian!

- separation of variables: $\psi(\vec{r})=X(x)\,Y(y)\,Z(z)\,T(t)$
  - end up with 4 eigenvalue equations!
  - quantize spatial eigenvalues by boundary conditions
  - obtain energy eigenvalue from TISE: $E_{lmn}=\frac{\hbar^2\|\vec{k}\|^2}{2m}+V(k_x,k_y,k_z)$
  - need to use co-ordinate system where V is separable
  - spherical co-ordinates for a central potential $V(r)$

$*$ example: free particle in 3d:

$$\hat{H}=\frac{\hat{p}^2}{2m}=\frac{-\hbar^2}{2m}\left(\partial_x^2+\partial_y^2+\partial_z^2\right)$$

$$X(x)=e^{ik_x x} \qquad Y(y)=e^{ik_y y} \qquad Z(z)=e^{ik_z z}$$

$$\psi(x,y,z)=\int d^3k\, A(\vec{k})\, e^{i\vec{k}\cdot\vec{r}}$$ 3-d Fourier transform

$$\hat{H}\Psi=\frac{-\hbar^2}{2m}\nabla^2\Psi=\frac{\hbar^2\|k\|^2}{2m}\Psi=E\Psi=\hbar\omega\Psi$$

$$\Psi(x,y,z,t)=\int d^3k\, A(\vec{k})\, e^{i\vec{k}\cdot\vec{r}-\omega t}$$