Propagation of unbound wave function, the free particle

Since V =0 for a free particle it's Schrodinger equation is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi,$$

A free particle can carry any positive energy tracking on the standard time dependent

exp (-iEt/h)

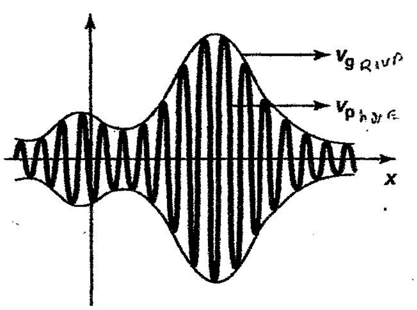
$$\Psi(x,t) = Ae^{ik(x-\frac{\hbar k}{2m}t)} + Be^{-ik(x+\frac{\hbar k}{2m}t)}.$$

The wave travels in the +-x direction at v = hk/2m, where x +- vt = constant.

The shape of a free wave does not change as it travels.

Eventually stationary states of the FP are propagating waves where their wave length $\lambda = 2\pi/k$ and according to deBroglie p = hk. The v = hk/2m. which is ½ the classical velocity v = (2E/m) to ½power.

The quantum mechanics wave function of a free particle travels at ½ the speed of the particle it represents.



V_g = PARTICLE

V_p = WAVE FUNCTION

$V_g = 2V_p$

The speed of the packet is group velocity = dw/dk. The phase velocity is w/k. The free wave function can not be normalized.

There is no such thing as a free particle with a define energy because separable solutions do not represent physically realizable states. Therefore the continuous variable k is use instead of the discrete index n.

$$\boxed{\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\phi(k)e^{i(kx-\frac{\hbar k^2}{2m}t)}\,dk.}$$

This equation carries a range of energies and speeds called a wave packet, normalized over ϕ(k).

The solution for the free particle is the above equation where :

$$\boxed{\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\Psi(x,0)e^{-ikx}\,dx.}$$

ϕ(k) acts like Cn.

The Scattering Matrix of an unbound particle

The solution to the left is:

$$\psi(x) = Ae^{ikx} + Be^{-ikx}, \quad \text{where } k \equiv \frac{\sqrt{2mE}}{\hbar}.$$

The solution to the right is:

$$\psi(x) = Fe^{ikx} + Ge^{-ikx}.$$

From this you can get the scattering matrix:

$$\begin{pmatrix} B \\ F \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} A \\ G \end{pmatrix}.$$

Where B and F are the outgoing amplitudes and A and G are the incoming amplitudes.

Scattering matrix for a free particle where V(x) =0 is:

$$S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

If V(x) is not 0

$$S = \begin{pmatrix} 2ir & 1 + 2ir \\ 1 + 2ir & 2ir^*(1 + 2it/1 - 2i^*t \end{pmatrix}$$

For a optical setup