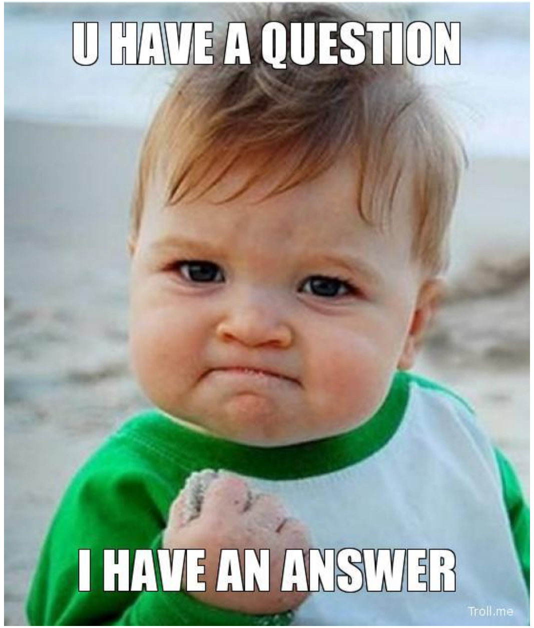
Fourier Analysis and Dispersion

Project by: Bowles, D.; Mathews, D.; Wu, J.

Introduction: What is Fourier Analysis?

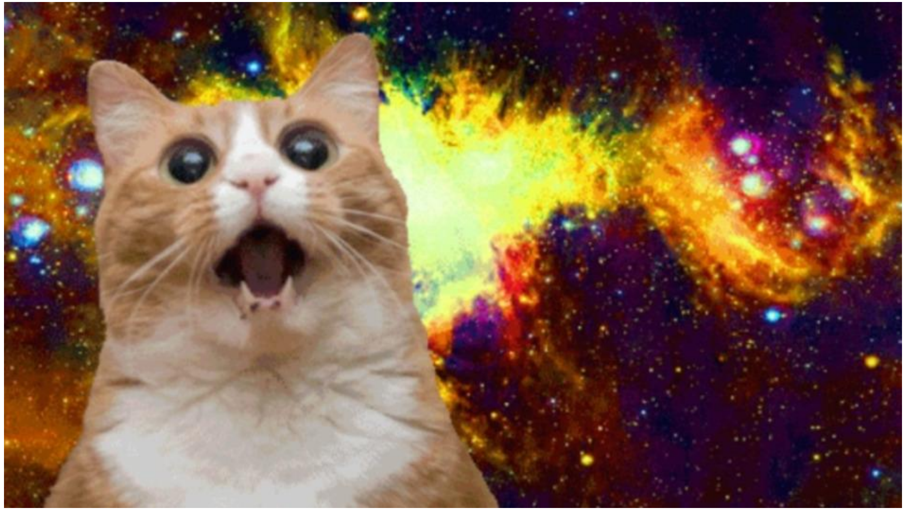


$$F(x) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi(x\frac{n}{N})}$$

$$f(n) = \frac{1}{N} \sum_{n=0}^{N-1} F(x) e^{j2\pi(x\frac{n}{N})}$$

Introduction: What is Fourier Analysis?

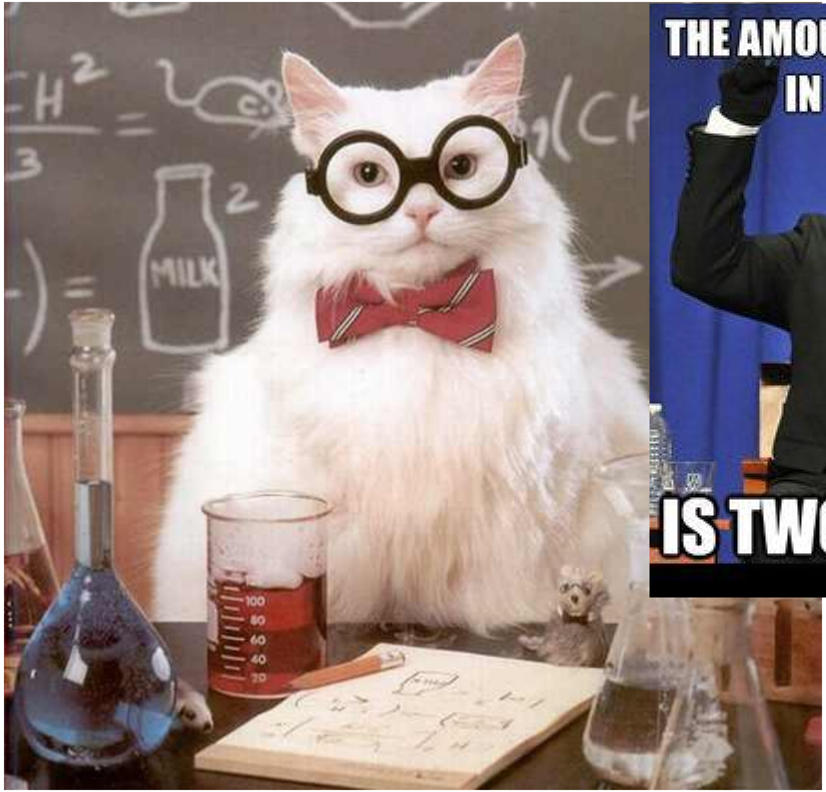
Energy Space gives you frequency and time

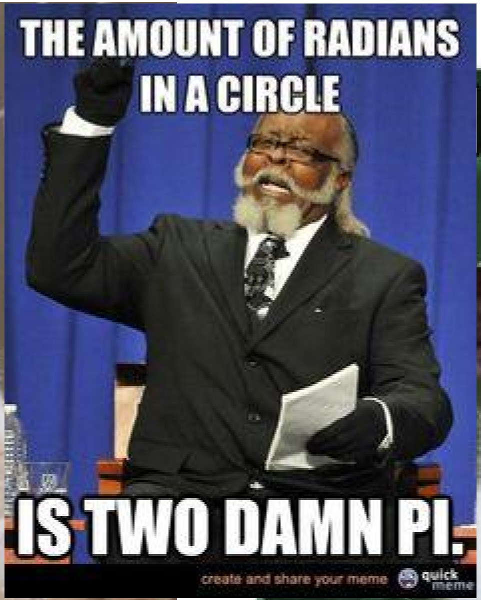


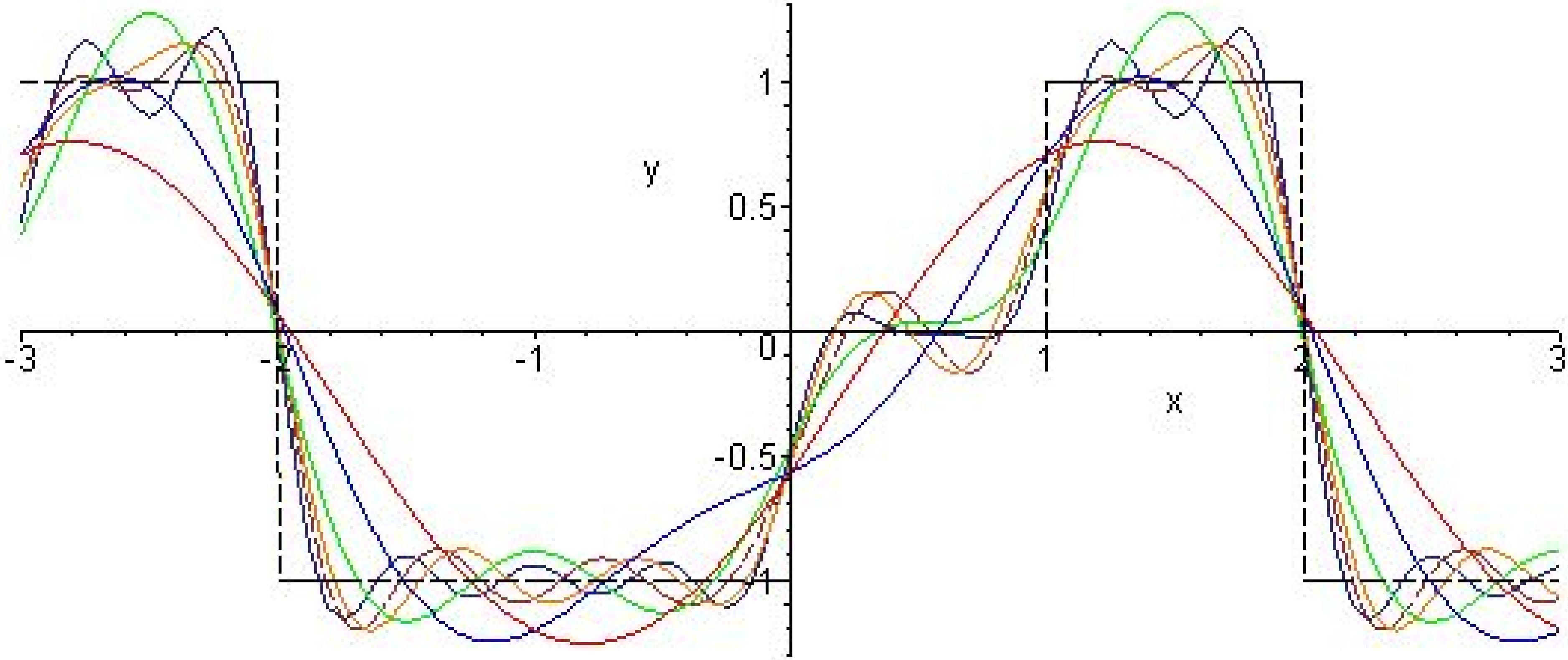
Introduction: What is Fourier Analysis?

Momentum space gives you momentum and position

Where did it come from?






y
1
0.5
0
-0.5
-1
-3
-2
-1
1
2
3
x

Where to start?

- A minimum uncertainty wavepacket in k space.
  - Has the general form exp(-α*(k-k_0)^2-i*ω(k)*t)
- Taylor expand ω(k) about the point k_0 .
  - ω(k) = ω(k_0) + dω/dk * (k-k_0) + d^2ω/dk^2 * ((k-k_0)^2) / 2
    - also note k= k_0+ k - k_0
- The derivatives of ω are evaluated at the point k_0 whose values are determined by the situation.

Methods of Space Transformation

- Fourier Transformation
  - means of changing from k space to x space
- Completing the square
  - To put the exponential into a gaussian form
- Contour integration of analytic functions
  - When completing the square the second term is complex, thus a change of variables shifts the original integral off the real axis into the imaginary plane.
  - Using Cauchy-Goursat theorem and taking limits at positive and negative infinity the problem simplifies to a Gaussian integral over all space.
    - Cauchy-Goursat Theorem: if a function is analytic through a simply connected domain then for every closed contour is zero in the domain

Methods of Space Transformation Continued

- Polar representation of complex numbers
    - This is used when simplifying the equation; specifically when dealing with complex number in the denominator to fractional powers. Representing complex numbers in terms of their polar form simplifies the forms and is necessary when calculating the real and imaginary components of the transformed form.