University of Kentucky, Physics 520
Homework #6, Rev. A, due Friday, 2015-10-07

0. Griffiths [2ed] Ch. 2 #9, #10, #11, #36, #41.

1. The **harmonic oscillator** is described using ladder opperators $a_\pm \equiv \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x \mp ip)$, which act on the stationary states of the TISE as follows: $a_+\psi_n = \sqrt{n+1}\psi_{n+1}$, $a_-\psi_n = \sqrt{n}\psi_{n-1}$.

a) Write out the matrices for a_- and a_+ in the hamonic oscillator energy eigenbasis $|n\rangle$ with components $\psi = c_0\psi_0 + c_1\psi_1 + c_2\psi_2 + c_3\psi_3 + \ldots$, (up to $n = 3$). *Hint: act $a_\pm$ on ψ to determine the new components. Then figure out the matrix that acts on $[c_0, c_1, c_2, c_3]^T$ in the same way.*

b) Verify that $a_+ = a_-{}^\dagger$ and $[a_-, a_+] = 1$.

c) Solve for x and p in terms of $a_\pm$ and calculate their matrices.

d) Calculate σ_x and σ_p to verify the Heisenberg uncertainty principle for the states ψ_0 and ψ_1.

e) Show that $[x, p] = i\hbar$, both algebraically and by matrix calculation. This the formal expression of the Heisenberg uncertainty principle.

f) Calculate the matrix for $H = p^2/2m + \frac{1}{2}mw^2x^2$. Why is it diagonal? Show that the uncertainty in energy σ_E is zero for any stationary state ψ_n. Note that by the Heisenberg uncertainty principle, an unperturbed particle will stay in this state forever.