L03-de Broglie matter wave, Bohr model of the atom

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de Broglie matter waves

* Plancks hypothesis led to the particle nature of waves. In particular, light, (electromagnetic waves) was quantized into packets of energy (particles) called photons: "γ"

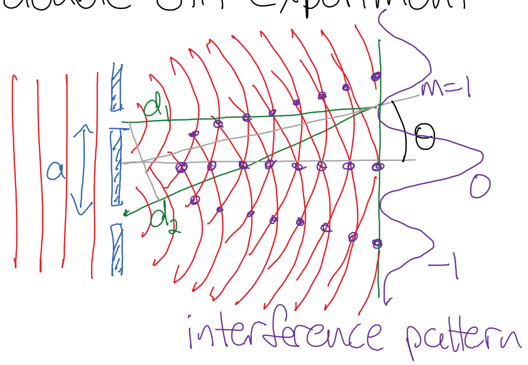
$$E = h\nu \quad \text{or} \quad E = \hbar\omega \quad \text{where} \quad \hbar = \frac{h}{2\pi} \quad \omega = 2\pi\nu \quad \text{(angular freq.)}$$

* de Broglie, guided by the principle of least action, completed the particle-wave symmetry by proposing that particles have an associated wavelength

$$\lambda = h/p \quad \text{or} \quad p = \hbar k \quad \text{where} \quad k = \frac{2\pi}{\lambda} \quad \text{(spatial frequency)}$$

* interference - a property of waves

- double slit experiment



phase difference

$$\phi = k(d_2 - d_1)$$

$$2\pi m = \frac{2\pi}{\lambda} a \cdot \sin(\theta)$$

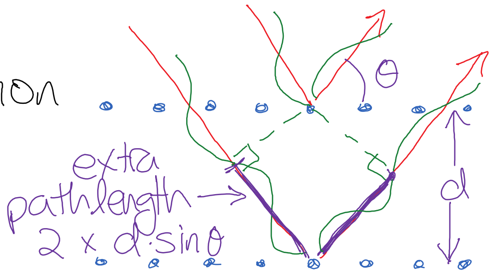
$$(n_2 \sin\theta_2 - n_1 \sin\theta_1) d = m\lambda$$

universal law for:
reflection, refraction, diffraction

$$kd = \frac{\omega}{c} \underbrace{nd}_{\text{optical pathlength}}$$

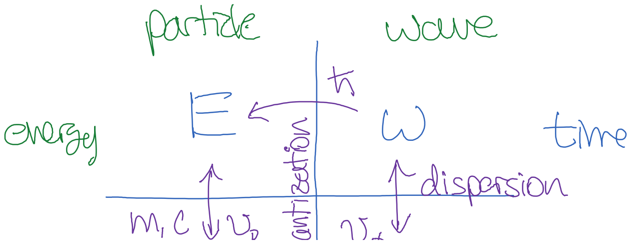
- Bragg formula: reflection from different planes in phase

$$m\lambda = 2d\sin\theta$$



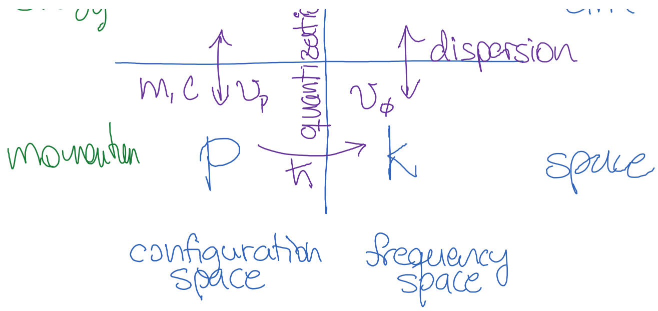
Bragg diffraction was discovered in electron scattering from a nickel crystal by Davisson & Germer

* Note the complementarity: BOTH matter and radiation exhibit both partcle & wave characteristics.



characteristics.

We summarize these relations with the following diagram:



Horizontal: quantization ...

$E = \hbar\omega$ of waves into packets of energy.

$p = \hbar k$ of particles into modes (energy levels)

Vertical: dispersion ... (Kinematics)

$E = \frac{1}{2}mv_p^2 = p^2/2m$ [NR] or $E^2 = (pc)^2 + (mc^2)^2$ [relativistic] free particle

$c = \lambda\nu$ or $\omega = kc$ phase velocity of photons v_ϕ

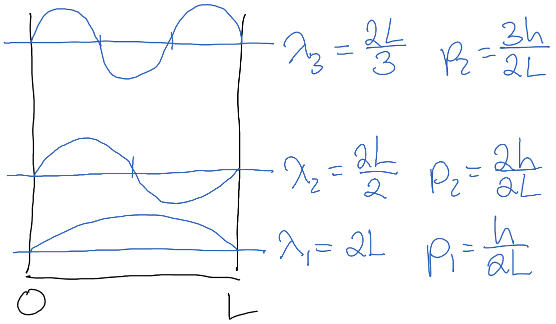
- For the massless photon, the diagram is consistent
  $(\omega = kc)\cdot\hbar$ $\quad \hbar\omega = \hbar k \cdot c$ $\quad E = pc$

- For massive particles, we will need a new dispersion relation for consistency: $E = \frac{p^2}{2m}$ $\quad \omega = \frac{\hbar}{2m}k^2$

We will build Schrödinger's equation from these quantization and dispersion relations.

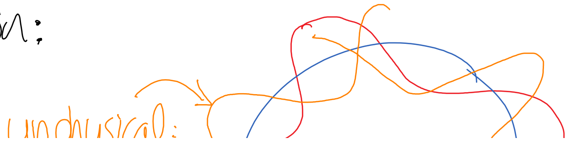
We will learn about the wave nature of particles by further examining dispersion.

* deBroglie's hypothesis leads to quantization of energy levels of matter (wavelength modes) by the application of propper boundary conditions.



Bohr model of the atom:

* deBroglie's hypothesis also gave physical intuition to Bohr's quantum condition:
The wave must wrap around on itself for a stable orbit



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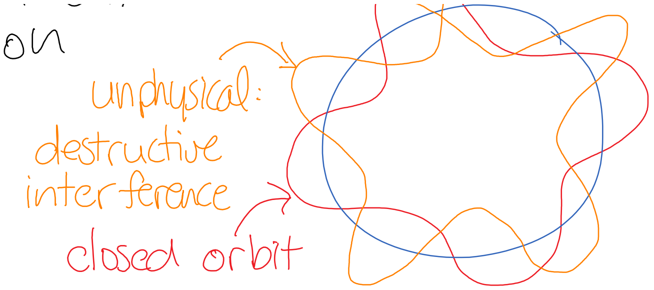

$$2\pi r = n\lambda = nh/p$$
$$L = |\vec{r}\times\vec{p}| = \hbar n$$

* Bohr's Postulates: over 10 years early, Bohr used both quantization of matter and radiation to explain the radiation spectrum of atomic hydrogen.

a) "stationary orbits": stable orbits of energy E_n satisfying the quantum condition: $L = n\hbar$

b) quantum transitions: "photon" $\frac{hc}{\lambda} = h\nu = E_{n_i} - E_{n_f}$

For circular orbits, $F_c = \frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$

$$v_n = \frac{Ze^2}{4\pi\varepsilon_0}\,\frac{1}{\underset{L=n\hbar}{\boxed{mvr}}} = \frac{Z}{n}\,\frac{e^2}{4\pi\varepsilon_0\hbar c}\cdot c = \frac{Z\alpha c}{n}$$

$Z < 137$! otherwise relativistic

$$r_n = \frac{Ze^2}{4\pi\varepsilon_0}\,\frac{1}{mv^2} = \frac{n^2}{Z}\underbrace{\underbrace{\frac{\hbar c}{m_e c^2}}_{\bar{\lambda}_c}\cdot\underbrace{\frac{4\pi\varepsilon_0\hbar c}{e^2}}_{1/\alpha}}_{a_0} = \frac{n^2}{Z}a_0$$

$$E_n = -\frac{Ze^2}{2\cdot 4\pi\varepsilon_0 r_n} = -\frac{Z^2}{n^2}\underbrace{\frac{m_e c^2}{2}\cdot\underbrace{\left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)^2}_{\alpha^2}}_{E_0} = -\frac{Z^2}{n^2}E_0$$

Thus $\frac{1}{\lambda} = \frac{E_{n_f} - E_{n_i}}{hc} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ $\quad R = E_0/hc = 109737/\text{cm}$

* Fundamental constants in natural units: eV

$kT = 25$ meV [300K] thermal energy

$e^2/4\pi\varepsilon_0 = 1.44$ eV·nm electric energy

$\hbar c = 197$ eV·nm [GeV·fm] quantum energy

$m_e c^2 = 0.511$ MeV mass energy

$\alpha = e^2/4\pi\varepsilon_0\hbar c = 1/137$ electric/quantum ratio

$E_0 = \frac{1}{2}m_e c^2\cdot\left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)^2 = 13.6$ eV ionization energy.

$a_0 = \bar{\lambda}_c/\alpha = \frac{\hbar c}{m_e c^2}\cdot\frac{4\pi\varepsilon_0\hbar c}{e^2} = 0.529$ Å Bohr radius 10Å=1nm