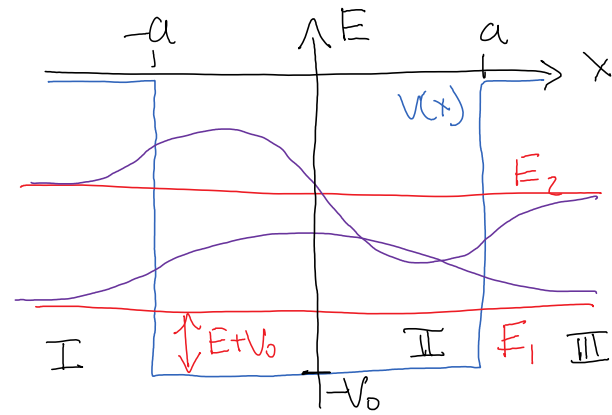


L21-Finite Square Well

Wednesday, November 4, 2015

08:27

- * potential. $V(x) = -V_0 \Theta(a - |x|)$
- finite: $V = 0$ if $|x| > a$
- square: V constant if $|x| < a$



- * Bound states:

3 regions need to be
sewn together w/ B.C.'s.

- can reduce to one boundary by symmetry:
 $V(x) = V(-x) \Rightarrow \Psi(x)$ either even or odd.

$$\Psi_I(x) = \pm \Psi_{III}(x)$$

$$\Psi_{II}(x) = D \cos(lx) \quad [\text{even}] \quad \text{OR} \quad C \sin(lx) \quad [\text{odd}]$$

$$\Psi_{III}(x) = F e^{-\kappa x}$$

obtain by symmetry.

[$e^{\kappa x}$ blows up as $x \rightarrow \infty$]

$$\frac{\hbar^2 l^2}{2m} = E + V_0$$

$$\frac{\hbar^2 \kappa^2}{2m} = E$$

B.C.'s:

EVEN

OR

ODD

$$i) \Psi_{II}(a) = \Psi_{III}(a): \quad D \cos(la) = F e^{-\kappa a} \quad \text{OR} \quad C \sin(la) = F e^{-\kappa a}$$

$$ii) \Psi'_{II}(a) = \Psi'_{III}(a): \quad -l D \sin(la) = -\kappa F e^{-\kappa a} \quad \text{OR} \quad l C \cos(la) = -\kappa F e^{-\kappa a}$$

solve for: i) $E \rightarrow l, \kappa$ and ii) D/F using B.C.'s i, ii
(normalize $\Psi(x)$ to get D and F individually).

$$\text{ratio: } \frac{\Psi'_{II}(a)}{\Psi_{II}(a)} = \frac{\Psi'_{III}(a)}{\Psi_{III}(a)}$$

$$\tan(la) = \frac{\kappa}{l} \quad \text{OR} \quad -\cot(la) = \frac{\kappa}{l}$$

$$\text{OR} \quad -\cot(z) = \frac{\sqrt{z_0^2 - z^2}}{z}$$

$$\Psi_{II}(a) = \overline{\Psi_{III}(a)}$$

$$\tan(z) = \frac{\sqrt{z_0^2 - z^2}}{z} \quad \text{OR} \quad -\cot(z) = \frac{\sqrt{z_0^2 - z^2}}{z}$$

where $z \equiv la$

$$\frac{\hbar^2 k^2}{2m} = E + V_0 = \frac{\hbar^2 \kappa^2}{2m} + V_0$$

$$(\kappa a)^2 = z_0^2 - (la)^2$$

$$z_0^2 = 2mV_0 a^2 / \hbar^2$$

$$E_n = \frac{\hbar^2}{2ma^2} z_n^2 - V_0$$

energy states.

`Plot[{Tan[z], -Cot[z], Sqrt[(52/z^2) - 1]}, {z, 0, 3 Pi}, PlotRange -> {-2, 7}]`

