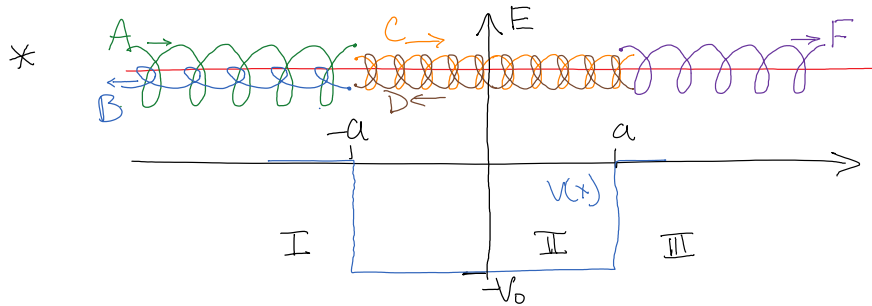


L22-Square Well Scattering

Wednesday, November 4, 2015 08:27



* Scattering States :

$$\Psi_I = Ae^{ikx} + Be^{-ikx}$$

$$\Psi_{II} = C\sin(lx) + D\cos(lx)$$

$$\Psi_{III} = Fe^{ikx} + Ge^{-ikx}$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$E + V_0 = \frac{\hbar^2 l^2}{2m}$$

(incoming from left if $G=0$)

B.C.'s: $x = -a$

$x = +a$

$$Ae^{-ika} + Be^{ika} = -C\sin(la) + D\cos(la)$$

$$ik(Ae^{-ika} - Be^{ika}) = l(C\cos(la) + D\sin(la))$$

$$C\sin(la) + D\cos(la) = Fe^{ika} + Ge^{-ika}$$

$$l(C\cos(la) - D\sin(la)) = ik(Fe^{ika} - Ge^{-ika})$$

$$\begin{pmatrix} 1 & 1 \\ ik & -ik \end{pmatrix} \begin{pmatrix} e^{-ika} A \\ e^{ika} B \end{pmatrix} = \begin{pmatrix} -S_l & C_l \\ C_l & S_l \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix}$$

$$\begin{pmatrix} S_l & C_l \\ C_l & -S_l \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ ik & -ik \end{pmatrix} \begin{pmatrix} e^{ika} F \\ e^{-ika} G \end{pmatrix}$$

$R = R^{-1} = R^T$

notes:

$$\begin{pmatrix} S_l & C_l \\ C_l & -S_l \end{pmatrix}^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} C_l & -S_l \\ S_l & C_l \end{pmatrix} = \begin{pmatrix} C_l & -S_l \\ S_l & C_l \end{pmatrix} = I$$

$$\begin{pmatrix} -S_l & C_l \\ C_l & S_l \end{pmatrix} \begin{pmatrix} S_l & C_l \\ C_l & -S_l \end{pmatrix} = \begin{pmatrix} C_l & -S_l \\ S_l & C_l \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} C_l & -S_l \\ S_l & C_l \end{pmatrix} = R_l R_l = R_{2l} = \begin{pmatrix} C_{2l} & -S_{2l} \\ S_{2l} & C_{2l} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ ik & -ik \end{pmatrix}^{-1} = \left[\begin{pmatrix} 1 & 1 \\ ik & -ik \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right]^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & -ik \\ 1 & ik \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -ik \\ 1 & ik \end{pmatrix}$$

thus

$$\begin{pmatrix} e^{-ika} A \\ e^{ika} B \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -ik \\ 1 & ik \end{pmatrix} \underbrace{\begin{pmatrix} -S_l & C_l \\ C_l & S_l \end{pmatrix} \begin{pmatrix} S_l & C_l \\ C_l & -S_l \end{pmatrix}^{-1}}_{R_{2l}} \begin{pmatrix} 1 & 1 \\ ik & -ik \end{pmatrix} \begin{pmatrix} e^{ika} F \\ e^{-ika} G \end{pmatrix}$$

$$= \begin{pmatrix} C_{2l} - i \frac{k^2 l^2}{2k^2} S_{2l} & i \frac{k^2 l^2}{2k^2} S_{2l} \\ -i \frac{k^2 l^2}{2k^2} S_{2l} & C_{2l} + i \frac{k^2 l^2}{2k^2} S_{2l} \end{pmatrix} \begin{pmatrix} e^{ika} F \\ e^{-ika} G \end{pmatrix}$$

$$= \begin{pmatrix} C_{2l} - i \frac{k^2 l^2}{2kl} S_{2l} & i \frac{k^2 l^2}{2kl} S_{2l} \\ -i \frac{k^2 l^2}{2kl} S_{2l} & C_{2l} + i \frac{k^2 l^2}{2kl} S_{2l} \end{pmatrix} \begin{pmatrix} e^{ika} \\ e^{-ika} \end{pmatrix}$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} (C_{2l} - i \frac{k^2 l^2}{2kl} S_{2l}) e^{2ika} & -i \frac{k^2 l^2}{2kl} S_{2l} \\ -i \frac{k^2 l^2}{2kl} S_{2l} & (C_{2l} + i \frac{k^2 l^2}{2kl} S_{2l}) e^{-2ika} \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix} = M \begin{pmatrix} F \\ G \end{pmatrix}$$

$$\begin{pmatrix} B \\ F \end{pmatrix} = \frac{1}{M_{11}} \begin{pmatrix} M_{21} & 1 \\ 1 & -M_{12} \end{pmatrix} \begin{pmatrix} A \\ F \end{pmatrix} = S \begin{pmatrix} A \\ F \end{pmatrix} \quad \left(\frac{k^2 l^2}{2kl} \right)^2 = 1 + \left(\frac{k^2 l^2}{2kl} \right)^2 S_{2l}^2$$

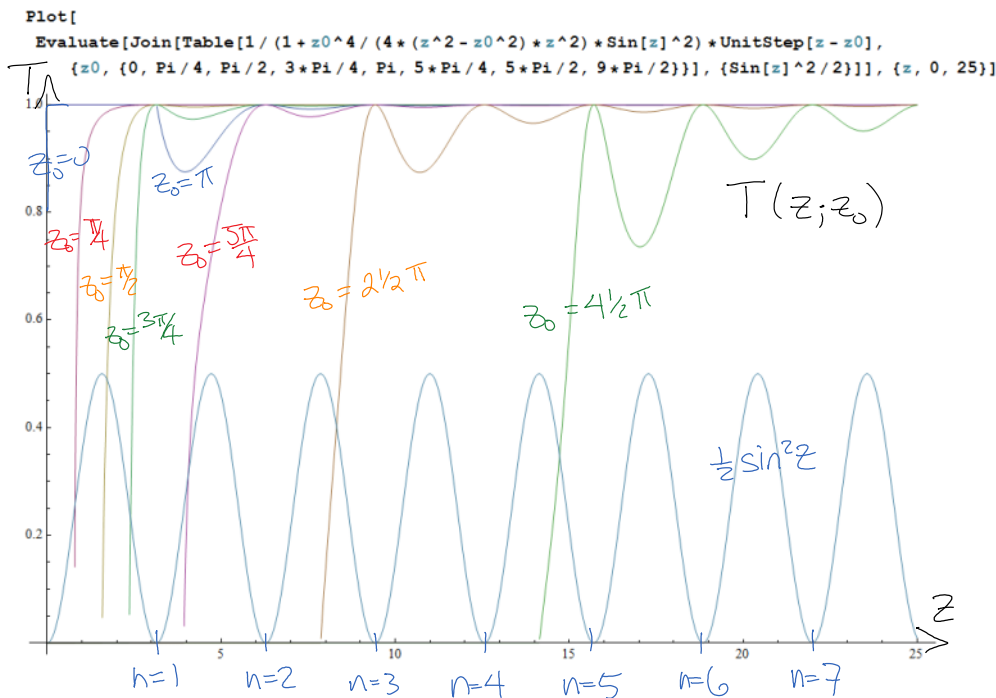
$\det M = 1$

$$T_l = \frac{|F|^2}{|A|^2} = |S_{21}|^2 = |M_{11}|^{-2} = [C_{2l}^2 + \left(\frac{k^2 l^2}{2kl} \right)^2 S_{2l}^2]^{-1} = [1 + \left(\frac{k^2 l^2}{2kl} \right)^2 S_{2l}^2]^{-1}$$

$$R_l = \frac{|B|^2}{|A|^2} = |S_{11}|^2 = |M_{21}|^2 T_l = \left(\frac{k^2 l^2}{2kl} \right)^2 S_{2l}^2 T_l = \left[\left(\frac{k^2 l^2}{2kl} \right)^2 S_{2l}^2 + 1 \right]^{-1}$$

$$T_l = \left[1 + \frac{V_0^2}{4E(E+V_0)} \sin^2 \left(\frac{2a}{\hbar} \sqrt{2m(E+V_0)} \right) \right]^{-1} = \left[1 + \frac{z_0^4}{4(z^2 z_0^2) z^2} \sin^2 z \right]^{-1}$$

$$T_l + R_l = \frac{1}{1+x} + \frac{x}{1-x} = 1 \quad \text{where } (z \equiv 2la)^2 = \frac{8ma^2(E+V_0)}{\hbar^2} \quad z_0^2 \equiv (2la)^2 - (2ka)^2 = \frac{8ma^2 V_0}{\hbar^2}$$



* resonances carry the same B.C.s from $-a \rightarrow a$ and have 100% transmission

