University of Kentucky, Physics 520
Exam 1, 2017-10-02

Instructions: This exam is closed book—you may not consult any person or reference material except your formula sheet. Show intermediate work for partial credit. [80 pts maximum]

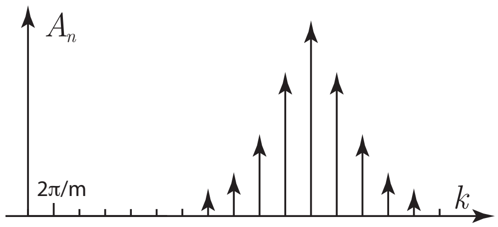
[20 pts] 1. Calculate the classical expectation value $\langle E\rangle$ of energy of single mode of blackbody radiation, assuming the continuous Boltzmann distribution of energy $P(E) \propto e^{-\beta E}$, with the constant parameter $\beta = 1/kT$ for a given temperature T.

Explain why the number of modes radiation between frequency ν and $\nu + d\nu$ in a cubic cavity is proportional to $\nu^2 d\nu$. What is the ultraviolet catastrophe?

[20 pts] 2. Classify the different categories, principles, and relations contained vertically and horizontally in the table $\begin{array}{c|c} E & \omega \\ \hline p & k \end{array}$ for a nonrelativistic particle of mass m and velocity v, and describe the physical effects. Show how these principles lead naturally to the Time Dependent Schrödinger Equation (TDSE) as a wave equation.

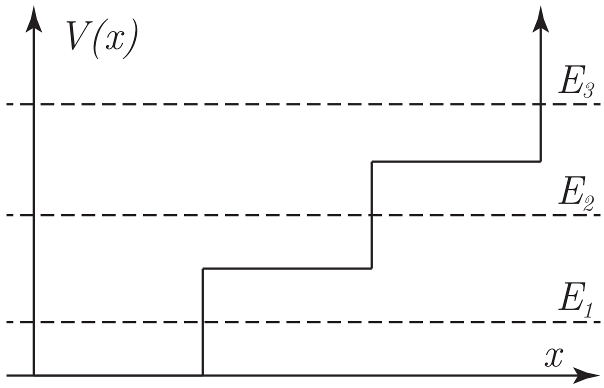
Compare and contrast classical waves versus particles. Explain how wave-particle duality leads to quantization of both particles [deBroglie] and waves [Planck]. Describe how matter behaves as both particles and waves in Quantum Mechanics.

[20 pts] 3. Given the following distribution A_n of component amplitudes of a free-particle wave, sketch the corresponding wave function $\psi(x)$. Identify the features associated with the a) fundamental frequency k_1, b) central frequency k_0, and c) bandwidth Δk in both A_n and $\psi(x)$.



Show that the wave function $\psi(x) = e^{ik_1 x} + e^{ik_2 x}$ is a pure carrier wave of frequency $\bar{k} = \frac{1}{2}(k_2 + k_1)$ modulated by a sinusoidal envelope of frequency $\hat{k} = \frac{1}{2}(k_2 - k_1)$. Assuming $\hbar\omega = \hbar^2 k^2/2m$, determine the phase and group velocity, and associate each with one of the features above.

[20 pts] 4. Sketch the three lowest energy eigenfunctions of the "staircase potential":



Calculate the normalized stationary state wave eigenfunctions $\psi_n(x)$ and energy eigenvalues E_n of the infinite square well potential $V(x) = 0$ if $0 < x < a$ and $V(x) = \infty$ otherwise. Sketch the first three wave functions and energy levels on a graph of the potential.