University of Kentucky, Physics 520
Homework #1, Rev. C, due Wednesday, 2017-09-05

0. Griffiths [2ed] Ch. 1 #1, #3, #13, #18. .

1. Maxwell-Boltzman distribution—The velocity spectrum of an ideal gas follows the Boltzman distribution $f(\boldsymbol{v}) = e^{-\frac{1}{2}mv^2/kT}$, assuming that the distribution of velocity states is uniform in (v_x, v_y, v_z) space and that the kinetic energy $T = \frac{1}{2}mv^2$ is thermally distributed.

a) Integrate the distribution $\int d^3v\, f(\boldsymbol{v})$ over θ and ϕ to obtain the Maxwell distribution $f(v)$ of the scalar velocity of the atoms in the gas. Normalize the distribution and calculate $\langle v \rangle$.

b) Show that if the density of atoms in a box is ρ, then the number of atoms escaping through a hole of area A per time is $\rho\langle v\rangle A/4$. Thus show that the average forward spectral intensity of black body radiation is $I = uc/4$, where u is the spectral energy density.

c) [bonus] An *effusive source* of gas is a nozzle in which the atomic mean free path is much longer the the width of the nozzle. Show that the velocity distribution is $f(\boldsymbol{v}) \propto v^3 \cos\theta\, e^{-\frac{1}{2}mv^2/kT}$. The forward-peaked velocity distribution makes an effusive source ideal for atomic beams.

2. Planck's law—The spectral energy density of thermal radiation from a black body is $u(\nu, T) = 8\pi h\nu^3/(c^3(e^{\hbar\nu/kT} - 1))$. You can solve the following problems using the functions `D`, `NSolve`, and `Integrate` in Mathematica.

a) Show that Plank's law obeys the following *scaling law*: that $u(\nu, T)/T^3$ depends only on $x = \hbar\nu/kT$, not ν or T individually, and thus, the graph of the spectral intensity $u(\nu)$ has the same shape for different temperatures. How should the ν- and u- axes scale with temperature to keep the graph invariant?

b) Show Wien's displacement law, that $\lambda_{max}T = b$ by calculating the value of x for which $u(\nu)$ attains its maximum at constant temperature T. Use this value to obtain the constant b.

c) Integrate $\int_0^\infty u(\nu, T)d\nu$ to obtain the total energy density over all wavelengths. Using 1c), calculate the constant σ_{SB} from the Stephan-Boltzman law $I = \sigma_{SB}T^4$.

3. Bohr's Oscillator. A 3-dimensional ideal frictionless harmonic oscillator of mass m and natural angular frequency ω behaves very similar to a hydrogen atom, except the central force law is $F = -kr$, with spring constant $k = m\omega^2$.

a) Use Bohr's quantization of angular momentum $L = \hbar n$ to calculate the energy levels E_n of the stationary states of this 'atom', and calculate spectrum of emitted wavelengths.

b) Show that Bohr's correspondence principle holds in this system.

c) [bonus] The classical Larmor formula for the power radiated by a point charge q under acceleration a is $P = (qa)^2/6\pi\epsilon_0 c^3$. Calculate the number of orbits an electron would make in the n^{th} orbital before decaying to the $(n-1)^{\text{th}}$ orbital.