University of Kentucky, Physics 520
Homework #5, Rev. A, due Monday, 2017-10-09

0. Griffiths [2ed] Ch. 2 #1, #2, #3, #4.

1. A **bouncy neutron** is trapped in the vertical z-direction on a perfectly reflecting horizontal neutron mirror ($V = \infty$ for $z < 0$) and by the earth's gravitational potential $V = mgz$ where m is the mass of the neutron and g is the acceleration due to gravity. Ignore the independent uniform horizontal motion in the x- and y-directions (see Nature **415** 299 (2002)).

a) Write down the Hamiltonian for this system and solve for the energy eigenstates. *Hint:* Substitute the dimensionless parameter $\zeta = z/z_0 + \zeta_n$ into the TISE and determine the constants z_0 and ζ_n in terms of m, g, $\hbar$, E to massage the TISE into the Airy equation, $d^2\psi/d\zeta^2 - \zeta\psi = 0$. This equation has two known independent solutions, the Airy functions $\mathrm{Ai}(\zeta)$ and $\mathrm{Bi}(\zeta)$. Quantize the energy by applying boundary conditions $\psi|_{z=0} = 0$ and $\psi|_{z\to\infty} \to 0$ to show that ζ_n is the n^{th} root of $\mathrm{Ai}(\zeta)$.

b) Calculate the quantum gravitational height scale z_0 [μm]. Calculate the total energy E_n [peV], frequency $\omega_n/2\pi$ [Hz], and the classical turning points z_n [μm] for the three lowest quantum states $n = 1, 2, 3$. Plot the energies E_n and wavefunctions $\psi_n(z)$ on the graph of $V(z)$ as usual.

c) Given the initial wave function $\psi_0(z) = 1/\sqrt{z_0}$ if $0 < z < z_0$ and 0 elsewhere, calculate the initial amplitudes of the first three energy states at $t = 0$, and at any later time t. Using these three states, calculate the expectation value of energy $\langle E \rangle$. What frequency should one vibrate the mirror in order to excite a neutron from the ground state to the first excited state?