University of Kentucky, Physics 520
Homework #8, Rev. A, due Wednesday, 2017-11-01

0. Griffiths [2ed] Ch. 2 #52, #53.

1. The **step potential** $V(x) = V_0\theta(x) = \{V_0 \text{ if } x > 0, \text{ or } 0 \text{ if } x < 0\}$ is the quantum mechanical analog of a wave traveling from a string of characteristic impedance Z_1 to another of impedance Z_2, joined at $x = 0$.

a) Calculate the transfer matrix for this potential.

b) Calculate the scattering matrix from the transfer matrix of part a).

c) Calculate the forward (coming in from the left) and backward (coming in from the right) probabilities of reflection and transmission for a particle of energy $E > 0$ using the elements of the scattering matrix of part b). *Hint: the forward transmission probability must take into account the difference of incoming and outgoing velocities:*

$$T_\ell = \left.\frac{j_F}{j_A}\right|_{G=0} = \left.\frac{k_2|F|^2}{k_1|A|^2}\right|_{G=0} = \frac{k_2}{k_1}|S_{21}|^2.$$

d) Show that the forward reflection coefficient is $R = 1$ if $E < V_0$. This is the analog of total internal reflection of light wave inside a fibre optic cable.

e) **[bonus]** What are the quantum mechanical analogs of the velocity v and impedance Z of a classical wave medium?

2. a) From H03 1d), the mechanical impedance of an oscillating mass is $Z = i\omega m$. We can also apply impedance to **elastic collisions**, interpreting the boundary conditions as conservation of momentum $p = mv$ and kinetic energy $E = \frac{1}{2}pv$, and defining impedance as $m = p/v$. For two particles of mass m_1 and m_2, initial velocities v_1 and v_2, and final velocities v_1' and v_2' confined to the x-axis, factorize the energy equation $\frac{1}{2}m_1(v_1^2 - v_1'^2) = \frac{1}{2}m_2(v_2'^2 - v_2^2)$ and use conservation of momentum to show that $v_1 + v_1' = v_2 + v_2'$, and therefore $\begin{pmatrix} v_1' \\ v_2' \end{pmatrix} = \frac{1}{m_1+m_2}\begin{pmatrix} m_1-m_2 & 2m_2 \\ 2m_1 & m_2-m_1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$. In particular, if two particles' impedances match, their velocities are exchanged during the collision [see Newton's cradle]. For $v_2 = 0$, show the that the *reflection coefficient* is $R \equiv E_2/E_0 = (m_1-m_2)^2/(m_1+m_2)^2$ and the *transmission coefficient* is $T \equiv E_2/E_0 = 4m_1m_2/(m_1+m_2)^2$.

b) **[bonus]** Apply the concept of impedance (moment of inertia) to conservation of angular momentum and energy: given an incoming baseball of mass m and velocity v, and a bat with moment of inertia I swinging at angular velocity ω, calculate the optimum radius r for transfer of energy from the bat to the ball.