L01-Distributions and Probability

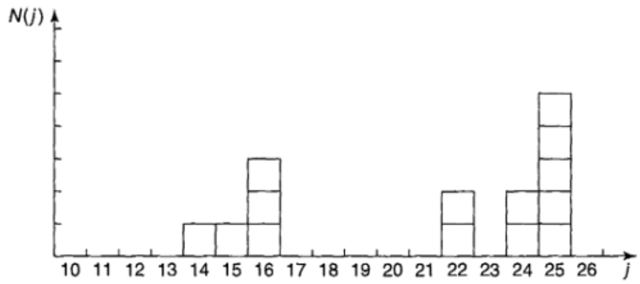
Monday, August 31, 2015 7:45 AM

* Distributions quantify how a quantity is "spread out"
  - "differential forms": everything after an integral or summation
  - "pseudo scalar": different from a Temperature "distribution"
  - distributed w/r some variable (age, position, wavelength).

1) discrete:

age distribution (year)

results from coin/die toss.



2) continuous: $Q = \int \rho(\vec{r})\, d\vec{r}$

$$\langle j \rangle = \frac{14+15+(16+16+16)+(22+22)+\ldots}{1+1+(1+1+1)+(1+1)+\ldots}$$

$$= \frac{14+15+3\cdot16+2\cdot22+2\cdot24+5\cdot25}{1+1+3+2+2+5}$$

$$= \frac{\Sigma N(j)\cdot j}{\Sigma N(j)}$$

mass density $\rho_m(\vec{r}) = dm/d\tau$

charge density $\rho_e(\vec{r}) = dq/d\tau$

energy density $u(\vec{r}) = \frac{1}{2}(\vec{D}\cdot\vec{E} + \vec{B}\cdot\vec{H})$

momentum density $\wp(\vec{r}) = \vec{D}\times\vec{B}$

* moments of a distribution

	MASS			CHARGE	
0th	$m = \int\rho(x)dx \cdot x^0$		Σm_i	$q = \int\rho(x)dx \equiv \int dq$	monopole
1st	$m\bar{x} = \int\rho(x)dx \cdot x^1$	or	$\Sigma m_i x_i$	$\vec{p} = \int\rho(x)dx\, \vec{r}$	dipole
2nd	$m\overline{x^2} = \int\rho(x)dx \cdot x^2$		$\Sigma m_i x_i^2$	$\overleftrightarrow{Q} = \int\rho(x)dx\, \frac{1}{2}(3\vec{r}\vec{r} - r^2 I)$	quadrupole

* cumulants (central moments)

$$\bar{x} = \frac{\int\rho(x)dx\cdot x}{\int\rho(x)} \quad \text{or} \quad \langle x \rangle$$

$$\langle f(x) \rangle = \frac{\int\rho(x)dx\, f(x)}{\int\rho(x)}$$ "weighted average"

$$\sigma_x = \Delta x = \underset{\text{deviation}}{\text{RMS}} = \sqrt{\frac{1}{n}\Sigma(x-\bar{x})^2}$$

$$\Delta f = \sqrt{\langle (f-\bar{f})^2 \rangle}$$ "standard deviation"

$$\text{or} \quad \langle (x-\bar{x})^2 \rangle = \frac{\int\rho(x)dx\,(x-\bar{x})^2}{\int\rho(x)dx}$$

$$\Delta f^2 = \langle f^2 \rangle - \langle f \rangle^2$$

or $\langle (x-\bar{x})^2 \rangle = \dfrac{\int p(x)\,dx\,(x-\bar{x})^2}{\int p(x)\,dx}$ $\quad \Delta f = \langle f^2 \rangle - \langle f \rangle^2$

proof: $\sigma_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2 = \frac{1}{n}\sum_n x_i^2 - 2x_i\bar{x} + \bar{x}^2 = \overline{x^2} - \bar{x}^2$

* probability distribution $\quad P(x) > 0 \quad \int P(x)\,dx = 1$ (normalized)

the "chance" of a particular random event occuring

ie a certain value of a random variable

the limit of a normalized histogram of occurrances as $n \to \infty$

* switching variables in a distribution:

$u_\nu(\nu)\,d\nu = u_\lambda(\lambda)\,d\lambda$ [invariant] $\qquad c = \nu\lambda \qquad \frac{d\lambda}{d\nu} = \frac{-c}{\nu^2}$

$u_\nu(\nu) = u_\lambda(\lambda)\left|\frac{d\lambda}{d\nu}\right| = u_\lambda(c/\lambda)\cdot\frac{c}{\nu^2}$ $\qquad (-)$: ν increases as λ decreases.