L03-Bohr's model of the atom

Friday, September 2, 2016 09:45

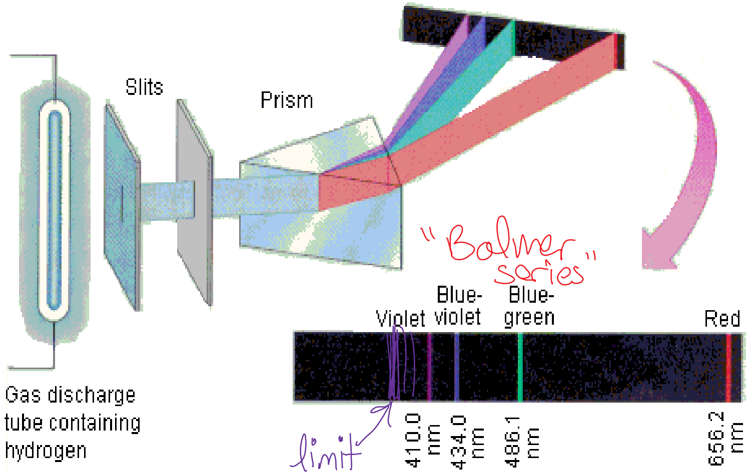
* last class we learned about black body spectra, today an equally puzzling effect, also a window into QM: atomic spectra

* spectral intensity is measured using the "dispersion" property of light waves, (wavelength dependence of velocity) to separate out the different wavelengths and separately measure their intensity. with a prism

Alternatively you can use the "diffraction" and "interference" effects to spread out the wavelengths with a diffraction grating.

Both of these effects will also be important in QM waves.

http://nothingnerdy.wikispaces.com/ATOMIC+SPECTRA+AND+ATOMIC+ENERGY+STATES



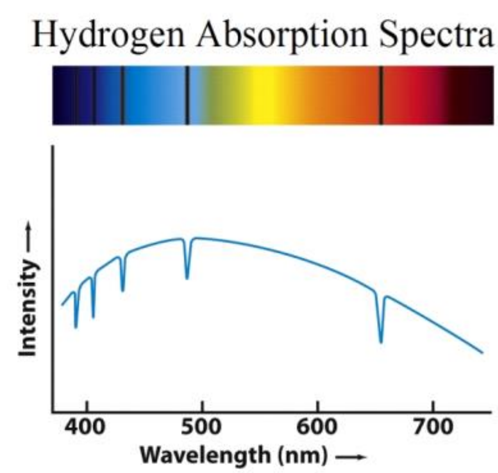


Figure 4-6ab
Discovering the Universe, Eighth Edition
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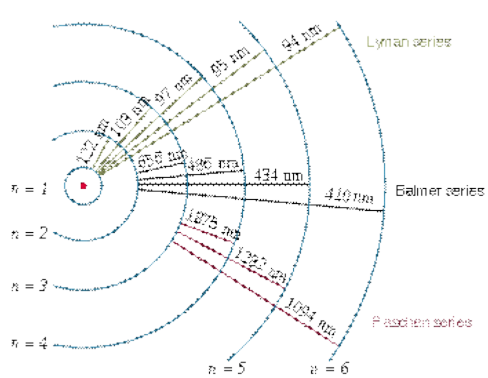
* Different series of specral lines were combined into a single formula for wavelength by Rydberg:

$$1/\lambda = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Lyman: ($n_f=1$, UV), Balmer ($n_f=2$, visible), Paschen ($n_f=3$, IR), Brackett ($n_f=4$), Pfund ($n_f=5$), Humphreys ($n_f=6$)



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* Earlier attempts to explain the lines as natural frequencies of oscillation of atoms in J.J. Thompson's "plum pudding" model of the atom failed. There was no physical model with this type of resonances. (example, system of masses & springs)

* Bohr's Postulates: over 10 years early, Bohr used both quantization of matter and radiation to explain the radiation spectrum of atomic hydrogen.

"reduced Planck's const" $\hbar \equiv \frac{h}{2\pi}$

a) "stationary orbits": stable orbits of energy E_n satisfying the quantum condition: $\boxed{L_e = \hbar n}$

b) quantum transitions: "photon" $E_{n_i} - E_{n_f} = \boxed{E_\gamma = h\nu} = \frac{hc}{\lambda}$

* We will review the Bohr atom "old quantum theory" because it has most of the elements of any quantum problem.

For circular orbits, $F_c = \frac{m v^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$

$$v_n = \frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{m_e v r} = \frac{Z}{n} \frac{e^2}{4\pi\varepsilon_0 \hbar c} \cdot c = \frac{Z\alpha c}{n}$$

$L = n\hbar$

$Z < 137$! (otherwise relativistic)

$$r_n = \frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{m_e v^2} = \frac{n^2}{Z} \overbrace{\underbrace{\frac{\hbar c}{m_e c^2}}_{\lambda_c} \cdot \underbrace{\frac{4\pi\varepsilon_0 \hbar c}{e^2}}_{1/\alpha}}^{a_0} = \frac{n^2 \lambda_c}{Z\alpha} = \frac{n^2}{Z} a_0$$

$$E_n = -\frac{Ze^2}{2 \cdot 4\pi\varepsilon_0 r_n} = -\frac{Z^2}{n^2} \underbrace{\frac{m_e c^2}{2} \cdot \underbrace{\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)^2}_{\alpha^2}}_{E_0} = -\frac{Z^2}{n^2} \frac{E_e}{2} \alpha^2 = -\frac{Z^2}{n^2} E_0$$

Thus $\frac{1}{\lambda} = \frac{E_{n_f} - E_{n_i}}{hc} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ $\quad R = E_0/hc = 109737/\text{cm}$

* Fundamental constants in natural units: eV

kT	$= 25$ meV [300 K]	thermal energy
$e^2/4\pi\varepsilon_0$	$= 1.44$ eV·nm	electric energy
$\hbar c$	$= 197$ eV·nm [GeV·fm]	quantum energy
$m_e c^2$	$= 0.511$ MeV	mass energy

$\alpha = e^2/4\pi\varepsilon_0 \hbar c \quad = 1/137 \quad$ electric/quantum ratio

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$E_0 = \frac{1}{2} m_e c^2 \cdot \left(e^2/4\pi\varepsilon_0\hbar c\right)^2 = 13.6\,\text{eV} \qquad$ ionization energy.

$a_0 = \bar{\lambda}_C/\alpha = \underbrace{\frac{\hbar c}{m_e c^2}}\cdot \frac{4\pi\varepsilon_0\hbar c}{e^2} = 0.529\,\text{Å} \qquad$ Bohr radius $\quad 10\,\text{Å} = 1\,\text{nm}$

$\bar{\lambda}_C = \frac{\hbar}{m_e c} = 386\,\text{fm} \qquad$ reduced Compton wavelength

* Correspondence Principle

Bohr did not have deBroglie's principle or $L = \hbar n$ to quantize the energy levels of the atom.

He solved for the energy levels that matched the Rydberg formula.

He formulated the correspondence principle to justify this selection of energy levels:

"The results of quantum physics match those of classical physics at large quantum numbers"

Ex: The orbital frequency $\approx$ radiation frequency as $n \to \infty$.

$$\omega_{cl} = \frac{v_n}{r_n} = \frac{Z\alpha c}{n} \cdot \frac{Z\alpha m_e c}{n^2\hbar} = \frac{(Z\alpha)^2 \cdot m_e c^2}{\hbar \quad n^3}$$

classical radiation at the orbital freq.

$$\omega_q = \frac{\Delta E}{\hbar} = \frac{Z^2}{\hbar} \cdot \frac{m_e c^2}{2} \cdot \alpha^2 \underbrace{\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)}_{\frac{(n^2+2n+1)-n^2}{n^2\cdot(n^2+2n+1)} \approx 2n} \xrightarrow{n\to\infty} \frac{(Z\alpha)^2 m_e c^2}{\hbar n^3}$$

Ex: Ehrenfest' theorem (expectation values classical)

$$\frac{d\langle x\rangle}{dt} = \left\langle \frac{\partial H}{\partial p} \right\rangle = \frac{\langle p \rangle}{m} \qquad \frac{d\langle p\rangle}{dt} = -\left\langle \frac{\partial H}{\partial x}\right\rangle = -\left\langle \frac{\partial V}{\partial x}\right\rangle$$

We'll prove these using Schrödinger's equation.