L04-de Broglie matter waves

Tuesday, August 23, 2016 08:12

* Plancks hypothesis led to the particle nature of waves. In particular, light, (electromagnetic wave) was quantized into packets of energy (particles) called photons: "γ"

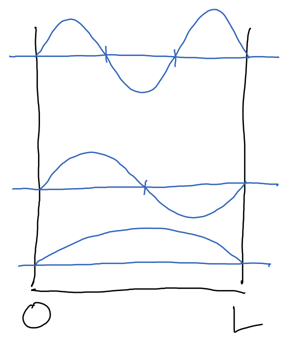
$$E = h\nu \quad \text{or} \quad E = \hbar\omega \quad \text{where } \hbar = \frac{h}{2\pi} \quad \omega = 2\pi\nu \text{ (angular freq.)}$$

* de Broglie, guided by the principle of special relativity, completed the particle-wave symmetry by proposing that particles have an associated wavelength

$$\lambda = h/p \quad \text{or} \quad p = \hbar k \quad \text{where } k = \frac{2\pi}{\lambda} \text{ (spatial frequency) or wave number}$$

http://aflb.ensmp.fr/LDB-oeuvres/De_Broglie_Kracklauer.pdf

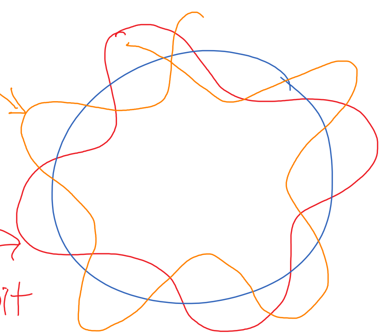
* de Broglie's hypothesis leads to quantization of energy levels of matter (wavelength modes) by the application of propper boundary conditions.



- we already saw this quantization in Rayleigh-Jeans "modes"

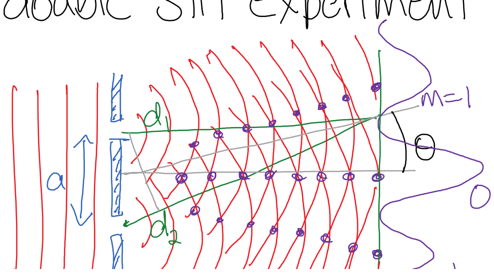
* de Broglie's hypothesis gave physical intuition to Bohr's quantum condition: The wave must wrap around on itself for a stable orbit

$$2\pi r = n\lambda = nh/p$$
$$L = |\vec{r} \times \vec{p}| = \hbar n$$



* interference - a property of waves

- double slit experiment



phase difference

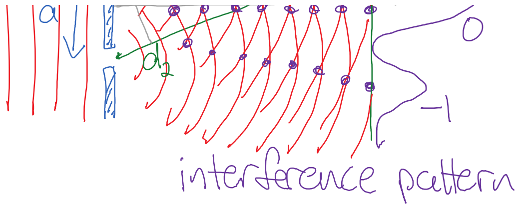
$$\phi = k(d_2 - d_1)$$

$$2\pi m = \frac{2\pi}{\lambda} a \cdot \sin(\theta)$$

$$(n_2 \sin\theta_2 - n_1 \sin\theta_1) d = m\lambda$$

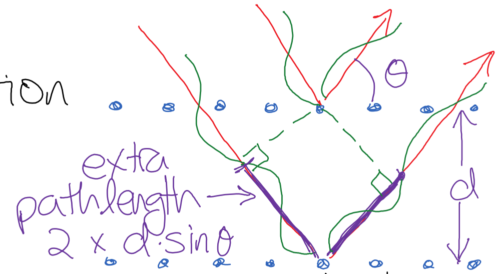
$$kd = \frac{\omega}{c} \underbrace{nd}_{\text{optical pathlength}}$$

universal law for: reflection, refraction, diffraction



universal law for:
reflection, refraction, diffraction

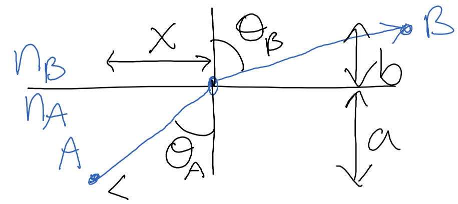
– Bragg formula: reflection from differant planes in phase



$$m\lambda = 2d\sin\theta$$

Bragg diffraction was discovered in electron scattering from a nickel crystal by Davisson & Germer

* Reconciliation of Fermat's and Maupertuis' principles:
– variational principles (Calculus of Variations): solve for the path which minimizes some line integral



a) Fermat's principle: light follows the path from A to B that minimizes the time to get there.

$$\Delta t = \frac{(x^2+a^2)^{1/2}}{c/n_a} + \frac{((l-x)^2+b^2)^{1/2}}{c/n_b} \qquad \frac{d}{dx}(c\Delta t) = \frac{x\, n_a}{\sqrt{x^2+a^2}} + \frac{-(l-x)n_b}{\sqrt{(l-x)^2+b^2}} = n_a\sin\theta_a - n_b\sin\theta_b = 0.$$

b) Maupertuis' principle: the true path of a system with generalized co-ordinates $\vec{q} = (q_1, q_2, q_3 \dots)$, going from $\vec{q}_A$ to $\vec{q}_B$ is an extremum of the abreviated action functional $S_0[q(t)] \equiv \int \vec{p}\cdot d\vec{q}$

where $p_i \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}^i}$ [generalized momentum] and $\mathcal{L} = T(q,\dot{q},t) - V(q,\dot{q},t)$ [Lagrangian]

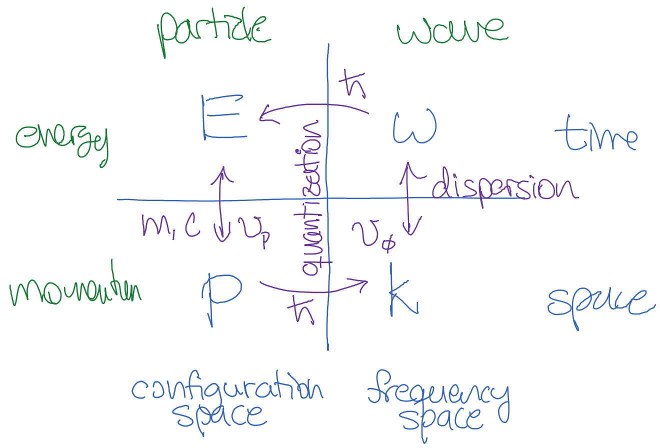
This can be generalized to Hamilton's principle of least action: $S \equiv \int \mathcal{L}\,dt$

The application of variational principles was too dificult for practical problems – a "wave equation" was needed to solve for standing waves.

* Note the complementarity: particle wave
BOTH matter and radiation

* Note the complementarity: BOTH matter and radiation exhibit both partcle & wave characteristics.

We summarize these relations with the following diagram:



Horizontal: quantization ...

$E = \hbar\omega$ of waves into packets of energy.

$p = \hbar k$ of particles into modes (energy levels)

Vertical: dispersion ... (kinematics)

$E = \frac{1}{2}mv_p^2 = p^2/2m$ [NR] or $E^2 = (pc)^2 + (mc^2)^2$ [relativistic] free particle

$c = \lambda\nu$ or $\omega = kc$ phase velocity of photons v_ϕ

- For the massless photon, the diagram is consistent
  $(\omega = kc)\cdot\hbar \quad \hbar\omega = \hbar k\cdot c \quad E = pc$

- For massive particles, we will need a new dispersion relation for consistency: $E = \frac{p^2}{2m} \quad \omega = \frac{\hbar}{2m}k^2$

We will build Schrödinger's equation from these quantization and dispersion relations.

We will learn about the wave nature of particles by further examining dispersion.