L31-Postulates of QM

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* what do you do in:

Classical mechanics? predict future measurements:
calculate (evolve) trajectory, (use conservation principles)

Quantum mechanics? predict what happens to a state
a) how it evolves in time undisturbed
b) what happens when you make a measurement

- note: a QM state is DEFINED by what you can measure!

* executive summary of measurement in Q.M.

1) observation is richer than classical $Q(x,p)$ due to complementarity
   it requires a Hermitian operator $\hat{Q}(x, -i\hbar\frac{\partial}{\partial x})$ on a state $|\Psi(x)\rangle$
2) every observable has special "determinate states" $|\phi_n\rangle \to q_n$
   these are eigenstates: $\hat{Q}|\phi_n\rangle = q_n|\phi_n\rangle$
3) any other state is a "superposition" of these (complex linear. combo)
   c_n = "probability amplitude"; $|\Psi\rangle$ is a complete set of amplitudes
4) observation is an irreversible projection - collapses the state
5) measurements with same eigenstates (commute) are compatible
6) otherwise the state evolves (a unitary rotation in $\mathcal{H}$ space)
   as the wave function propagates under the T.I.S.E. with $\mathcal{H}$
7) Dirac notation treats these opperations in a unified framework

* Practically, what do we do to solve problems in QM?

a) calculate eigenfunctions of operators (Hermitian)
b) rotate vectors or change basis (Unitary)

- solve $\hat{\mathcal{H}}|\psi_n\rangle = E_n|\psi_n\rangle$ to get stationary states.
- change basis of initial state to $|\Psi_0\rangle = \sum_n c_n|\psi_n\rangle$ energy basis

- rotate stationary states in time: $|\Psi(t)\rangle = \sum_n c_n |\psi_n\rangle e^{-iE_n t/\hbar}$
- solve eigenfunctions of an observable $\hat{Q}|\phi_n\rangle = q_n|\phi_n\rangle$
- rotate basis to observable states: $|\Psi(t)\rangle = \sum_n a_n(t)\,|\phi_n\rangle$ to calculate probability $|a_n|^2$ of measuring q_n
- project state (collapse wavefn) $|\psi\rangle \to |\phi_n\rangle$ after measuring q_n

What are the tools? inner product $\langle\psi|\phi\rangle$ & orthogonal basis
- orthonormality: $\langle\phi_n|\phi_m\rangle = \delta_{nm}$ closure: $\sum_n |\phi_n\rangle\langle\phi_n| = I$

* Postulates of QM:

1) Hilbert space of states: superposition postulate
State $|\psi\rangle$ = collection of complex probability amplitudes $\psi(x)$ or c_n (vector components) which linearly combine to form new states
It has an inner product $\langle\psi|\psi\rangle = \int dx\,|\psi(x)|^2 = \sum_n |c_n|^2$ and is normalizable $\langle\psi|\psi\rangle = 1$ so probabilities add to 100%

2) Hermitian observables: expansion/projection postulate
observable = collection of determinate states & measurements (operator $\hat{Q}$) with real eigenvalues q_n & orthogonal eigenvectors $|\phi_n\rangle$ which form a complete set of basis vectors: $|\psi\rangle = \sum_n a_n|\phi_n\rangle$
- $|a_n|^2$ = probability of measuring q_n
- $|\psi\rangle \to |\phi_n\rangle$ after measuring q_n

3) Hamiltonian: evolution postulate
states evolve in time according to Schrödinger Eq: $\hat{H}|\Psi\rangle = \hat{E}|\Psi\rangle$
- eigenstates of $\hat{H}$ are stationary states: $\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle$
- evolution of mixed states: $|\Psi(x,t)\rangle = \sum_n c_n|\psi_n\rangle e^{-iE_n t/\hbar}$

4) Heisenberg: uncertainty postulate
- canonical commutation relation $[\hat{x},\hat{p}] = i\hbar$ for conjugate observables
- position and momentum are complementary: $\Delta x\,\Delta p \geq \hbar/2$

- momentum operator $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ and eigenstates $\psi_p(x) = e^{ipx/\hbar}$

5) Pauli: exclusion postulate
- identical particle exchange symmetry $\psi(x_1, x_2) = \pm\psi(x_2, x_1)$
- only one fermion can occupy each state

+: bosons $s = 0, 1, 2, \ldots$
−: fermions $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots$

(will discuss next semester)