**University of Kentucky, Physics 521**
**Homework #12, Rev. B, due Wednesday, 2016-03-02**

**0.** Griffiths [2ed] Ch. 5 #2, #7, #15, #17, #35, #36.

**1. Davilov Parameters** [Danilov, Phys. Lett. **35B**, 579 (1971)] The key properties of parity violation in elastic nn, np, and pp scattering (neutrons and protons) can be deduced from basic symmetry and conservation principles studied in class.

The neutron and proton are considered as two states of a single particle, the *nucleon* 'N', just as their constituent up 'u' and down 'd' particles are both quarks 'q'. In analogy with the two-state system spin, we say the nucleon has *isospin* $I = \frac{1}{2}$. The proton ($I_3 = \frac{1}{2}$) and neutron ($I_3 = -\frac{1}{2}$) are both represented by two-component *isospinors* $\Upsilon = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, respectively. Their operators are Pauli matrices writen $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ as opposed to $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. Because spinors and isospinors live in different spaces, we use the symbols $I$ and $\tau$ for isospin, and use generic indices $(1, 2, 3)$ instead of spatial coordinates $(x, y, z)$. Addition of isospin works exactly the same as spin—in particular, two-particle systems have the symmetry of an $I = 0$ singlet (for example, the $\omega$ meson) and $I = 1$ triplet (the $\rho^+, \rho^0, \rho^-$ mesons, which are identical except for charge).

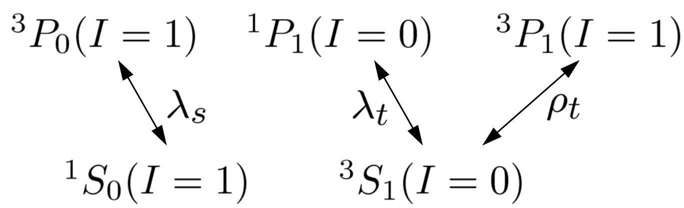
**a)** For now, neglect the spatial part of the nucleon wavefunction $\psi(\boldsymbol{r}, \boldsymbol{S}, \boldsymbol{I}) = \phi(\boldsymbol{r})\chi_{m_S}\Upsilon_{m_I}$. Write down the spin-isospinor $\chi\Upsilon$ for the four different spin and isospin combinations of N. Denote the two spin states by $\uparrow$ and $\downarrow$, and the isospin up and down states by p and n, respectively.

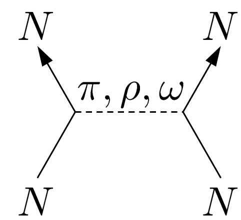
**b)** Construct all 10 symmetric and 6 antisymmetric NN wavefunctions by [anti]symmetrizing the 16 combinations of two single-particle spin-isospinors. Show that each of these states can be written as a linear combination of states with definite total spin and isospin, ie., the product of a spin singlet or triplet multiplied by an isospin singlet or triplet). What is the particle exchange symmetry of each? [**bonus**]: How many completely symmetric and antisymmetric states are there in the NNN system? Practical examples are the p, n, $\Delta^{-,0,+,++}$ states of the three-quark system qqq.

**c)** Transform the two-particle spatial wave function $\phi_1(\boldsymbol{r}_1)\phi_2(\boldsymbol{r}_2)$ into CM-relative coordinates $\phi_{\boldsymbol{R}}(\boldsymbol{R})\phi_{\boldsymbol{r}}(\boldsymbol{r})$ about the center of mass and reduced mass. Show that $P_{12}\phi = P_{\boldsymbol{r}}\phi$, where $P_{12}$ is the normal particle exchange operator and $P_{\boldsymbol{r}}$ is parity (spatial inversion) acting on the relative coordinate: $\boldsymbol{r} \to -\boldsymbol{r}$. As free particles, the two spatial wavefunctions (either particle 1,2 or $\boldsymbol{R}, \boldsymbol{r}$) have definite orbital angular momentum with quantum numbers $\ell, m_\ell$. Disregarding the radial dependence (in low-energy scattering), show that the relative wave function $Y_{\ell m}(\hat{\boldsymbol{r}})$ is even[odd] under $P_{\boldsymbol{r}}$ for even[odd] $\ell$, so that the simplest parity violating amplitudes are S-P ($\ell = 0 \to 1$) transitions.

**d)** The total wave function $\psi = \phi\chi\Upsilon$ must be antisymmetric under particle exchange $P_{12}$ by the Pauli exclusion principle. Mix even/odd combinations of $Y_{\ell m}(\hat{\boldsymbol{r}})$ from part c) and $\chi\Upsilon$ from part b) to construct all S and P states. Use linear combinations of these to create eigenfunctions of total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$. Denote each state in spectroscopic notation ${}^{2S+1}L_J(I)$, where $S$,$L$,$J$, and $[I]$ are the total [iso]angular momentum quantum numbers of the combined NN system.

Using conservation of total angular momentum $\boldsymbol{J}$, we find there are only five allowed S-P transition amplitudes: $\lambda_s(\Delta I = 0, 1, 2)$, $\lambda_t(I = 0)$, and $\rho_t(I = 0 \to 1)$, as illustrated below.




**e) [bonus]** In the DDH meson exchange model [Desplanques, Donoghue, Holstein, Ann. Phys. **124**, 449 (1980)], the spin and isospin dependence of the parity violating NN interaction is characterized by the properties of physical mesons ($\pi$, $\rho$, $\omega$), which transmit the force. Ignoring the lightest meson, the long-range pion $\pi$, correlate the five Danilov elastic transition amplitudes with the five meson exchange amplitudes $h_\rho^{\Delta I=0,1,2}$ and $h_\omega^{\Delta I=0,1}$.