University of Kentucky, Physics 521
Homework #13, Rev. A, due Monday, 2016-04-04

0. Griffiths [2ed] Ch. 5 #29, Ch. 6 #1, #5, #16, #36.

1. Hyperfine Zeeman splitting in deuterium is similar to hydrogen, except that the nucleus is a spin-1 deuteron instead of a spin-$\frac{1}{2}$ proton. The 1s ground state has 6-fold degeneracy, neglecting hyperfine structure. Since the deuteron is a loosely bound proton–neutron isosinglet dominated by $L = 0$, its magnetic moment is approximately the sum of its constituents $\mu_d = g_I \mu_N = 0.857\ \mu_N \approx \mu_p + \mu_n = 2.79\ \mu_N - 1.91\ \mu_N$.

a) Using the Pauli exclusion principle, show that the deuteron must be a spin triplet ($S = 1$).

b) Show that the hyperfine perturbation to the Hamiltonian is $\mathcal{H}'_{\rm hf} = b\vec{\boldsymbol{I}}\cdot\vec{\boldsymbol{S}}$ and calculate the b for deuterium. Show that the perturbation for the Zeeman effect is $\mathcal{H}'_{\rm Z} = (g_S\mu_B\vec{\boldsymbol{S}}/\hbar + g_I\mu_N\vec{\boldsymbol{I}}/\hbar)\cdot\vec{\boldsymbol{B}}_{ext}$, ignoring $\vec{\boldsymbol{L}}$. Compare the magnitude of g_S and g_I for the deuterium. Why can we ignore the fine structure when considering degeneracy of the 1s states?

c) Using degenerate perturbation theory, calculate the hyperfine energy shift for deuterium in a weak external magnetic field $B_{ext} \ll B_{int}$. What are the eigenstates and good quantum numbers which break the degeneracy of the Bohr levels? Which quantum numbers are still degenerate? Calculate g_F for the Zeeman shift $(g_F\mu_B\vec{\boldsymbol{F}}/\hbar)\cdot\vec{\boldsymbol{B}}_{ext}$, the field $B_{int} = b\hbar/g_F\mu_B$ where the two perturbations are approximately equal, and plot the Zeeman shift as a function of B_{ext}.

d) In a large external field, $B_{int} \ll B_{ext}$, we must first break the degeneracy according to the Zeeman shift, which dominates hyperfine structure. Calculate the perturbed energies as a function of B_{ext}. What are the good quantum numbers and corresponding eigenstates? What degeneracies remain? Use these states to calculate the hyperfine energy shift (constant for B_{ext}).

e) In intermediate fields where both perturbations are of the same order, we must diagonalize $\mathcal{H}'_{\rm hf} + \mathcal{H}'_{\rm Z}$ together. Calculate this 6×6 matrix in the $nlm_lm_sm_I$ basis and diagonalize it to obtain the exact energy shifts and plot them as a function of B_{ext}. Show that the small- and large-field limits match (b) and (c) respectively. Repeat the calculation and diagonalization of matrix elements in the $nljFM_F$ basis to show that the result is independent of basis.