L70-Hyperfine structure

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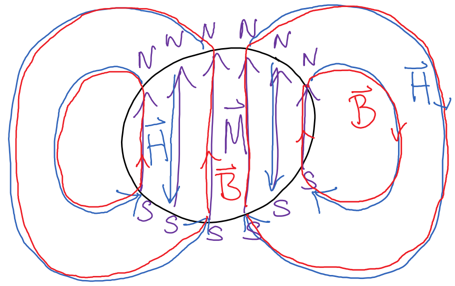
* Dipole energy: $H' = -\vec{\mu}_e \cdot \vec{B}$

- Zeeman effect: $\vec{B}_{ext}$ external field
- fine structure spin-orbit coupling: $\vec{B} = \frac{\mu_0 I}{2r} = \frac{1}{4\pi\epsilon_0}\frac{e}{mc^2 r^3}\vec{L}$
- hyperfine structure:

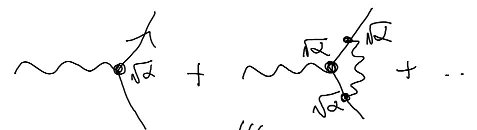
$$\vec{H}_{ext} = -\nabla U = \frac{\vec{\mu}\cdot\vec{r}}{4\pi r^3} \qquad \vec{H}_{int} = -\frac{1}{3}\vec{M}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\left(3\hat{r}\,\hat{r}\cdot\vec{\mu} - \vec{\mu}\right) + \frac{2}{3}\mu_0\,\vec{\mu}\,\delta^3(\vec{r})$$



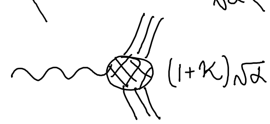
* Magnetic moment $\vec{\mu}_{e,p}$ of the electron & proton:

$$\mu_e = g_e \mu_B \vec{S}/\hbar \quad g_e = 2.00232 \approx 2\underbrace{\left[1 + \frac{\alpha}{2\pi} + O(\alpha^2)\right]}_{a} = $$



nuclear spin

$$\mu_p = g_p \mu_N \vec{I}/\hbar \quad g_p = 5.59 \quad \text{ie} \quad \mu_p = \frac{1}{2}g_p\mu_N = \underbrace{2.79}_{1+\kappa_p}\mu_N$$



$\mu_B = \frac{e\hbar}{2m_e}$ Bohr magneton, $\mu_N = \frac{e\hbar}{2m_p}$ Nuclear magneton

* spin-spin coupling:

$$\mathcal{H}'_{hf} = -\mu_e\cdot\vec{B} = +g_e\mu_B/\hbar\,\vec{S}\cdot g_p\mu_N/\hbar\left[\frac{\mu_0}{4\pi}\left(3\hat{r}\,\hat{r}\cdot\vec{I} - \vec{I}\right) + \frac{2\mu_0}{3}\vec{I}\,\delta^3(\vec{r})\right]$$

$$= \frac{\mu_0 g_e g_p e^2}{16\pi m_e m_p}\left\{\underbrace{\left(3\,\vec{S}\cdot\hat{r}\,\hat{r}\cdot\vec{I} - \vec{S}\cdot\vec{I}\right)}_{\text{dipole field}} + \underbrace{\frac{8\pi}{3}\vec{S}\cdot\vec{I}\,\delta^3(\vec{r})}_{\text{contact term}}\right\}$$

$$E'_{hf} = \langle \mathcal{H}'_{hf}\rangle$$

integrates to 0 over the sphere when $l=0$

also $|\psi_{100}(0)|^2 = \frac{1}{\pi a^3}$

$$= \frac{\mu_0 g_e g_p e^2}{6\pi m_e m_p a^3}\langle \vec{S}\cdot\vec{I}\rangle$$

looks familiar!

$$a = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = \frac{4\pi\hbar^2}{\mu_0 m_e c^2 e^2}$$

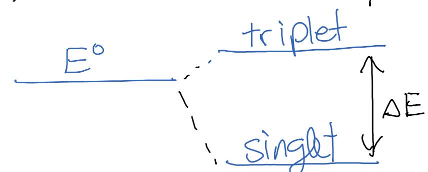
- use the same trick and couple nuclear & atomic spin

$(\vec{F} = \vec{I} + \vec{J})^2$ note if $L=0$ then $\vec{J} = \vec{S}$

$$F(F+1) = I(I+1) + S(S+1) + 2\vec{S}\cdot\vec{I}/\hbar^2$$

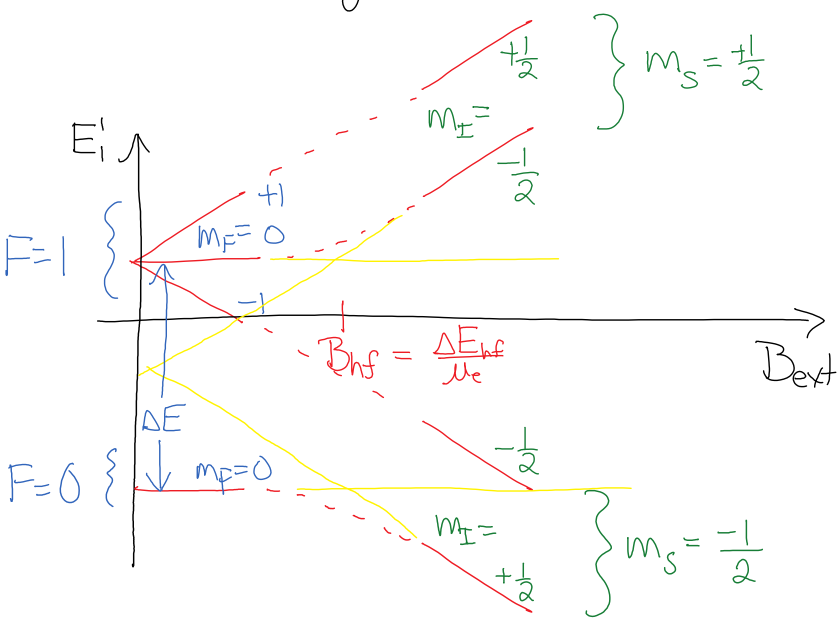
$I = 1/2$, $S = 1/2$ $\quad F = \begin{cases} 1 & \text{triplet } \uparrow\uparrow,\ \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow),\ \downarrow\downarrow \\ 0 & \text{singlet } \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \end{cases}$ $\quad \frac{\vec{S}\cdot\vec{I}}{\hbar^2} = \begin{cases} \frac{1}{4} \\ -\frac{3}{4} \end{cases}$

thus $E^1_{hf} = \underbrace{\frac{2\hbar^4 g_e g_p}{3 m_e^2 m_p c^2 \cdot a^4}} \begin{cases} 1/4 & F=1 \\ -3/4 & F=0 \end{cases}$



$\Delta E_{hf} = 5.88\,\mu eV \qquad \nu = \frac{\Delta E_{hf}}{h} \approx 1420\ \text{MHz}$

* Zeeman splitting of hyperfine structure:



if $B \ll B_{hf}$

then F, m_F are good quantum #'s

if $B \gg B_{hf}$

then m_S, m_I are good quantum #'s.