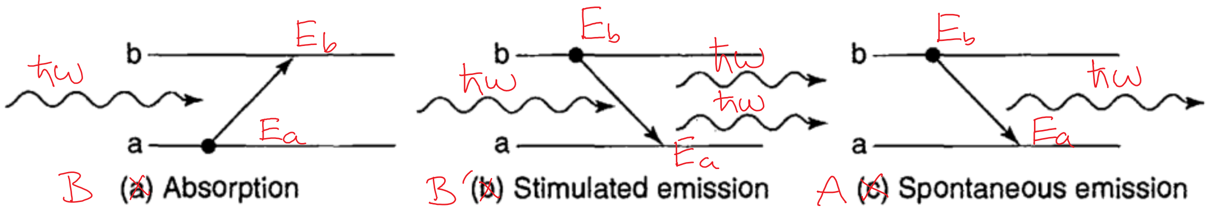
L76-Emission and Absorption of Radiation

Wednesday, April 6, 2016 08:47

$\ast$ Absorption, Stimulated Emission, Spontaneous emission.



B) absorption $E_a + \hbar\omega_0 \rightarrow E_b$ strongly peaked at $\omega \approx \omega_0$ (photon)

B′) stimulated emission $E_b \rightarrow E_a + \hbar\omega$ same probability!

- Recall "Rabi flopping" between spinstates in osc. field exact solution, not just perturbative.

- Non classical: predicted by Einstein in 1917 by comparing emission/absorption of black body radiation (Planck, 1900). Predicted ~ 10yrs before Heisenberg/Schrödinger eq! (1926)

- Laser (Light Amplification by Stimulated Emission of Radiation) uses this process to amplify coherent radiation, requires:

  i) population inversion: so that $b \rightarrow a$ dominates $a \rightarrow b$ must "pump" atoms from $a \rightarrow b$ state

  ii) cavity: multiple passes of photon enhances stimulated emission and sharpens wavelength of laser.

A) spontaneous emission

- classical process would be forbidden quantum-mechanically without a perturbation.

- in QED, the vacuum (ground state) includes "zero-point radiation", like $E_0 = \frac{1}{2}\hbar\omega$ ground state of harmonic oscillator.
  problem #9.9

- zero pt. radiation responsible for "Casimir force" between parallel plates

- thus Quantum Mechanics: ALL radiation is stimulated
  vs. Classical Mechanics: ALL radiation is spontaneous.

- also "thermally stimulated emission", stimulated by black body radiation, at low frequencies $\omega \ll$ THz (300K)
  problem #9.8

2016-04-06

* Monochromatic EM waves

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \approx \vec{E}_0 \cos\omega t \quad \text{if} \quad \vec{k}\cdot\vec{r} \ll 2\pi \quad \text{ie.} \quad \frac{2\pi}{k} = \lambda \gg a$$

$$\mathcal{H}' = -\hat{\wp}\cdot\vec{E} = -q\,\vec{r}\cdot\vec{E}_0 \cos\omega t \quad \text{so} \quad V(\vec{r}) = -\hat{\wp}\cdot\vec{E}_0 \quad \text{operator}$$

$$\mathcal{H}_{ab} = \langle\psi_a|\mathcal{H}'|\psi_b\rangle = -\vec{\wp}\cdot\vec{E} = V_{ab}\cos\omega t \qquad V_{ab} = -\vec{\wp}\cdot\vec{E}_0$$

where $\vec{\wp} = \langle\psi_a|\hat{\wp}|\psi_b\rangle = q\langle\psi_a|\vec{r}|\psi_b\rangle$

$$P_{\vec{r}}\,\psi_{lm}(\vec{r}) \equiv \psi_{lm}(-\vec{r}) = \underbrace{(-1)^l\,\psi_{lm}(\vec{r})}_{\text{either even or odd}} \quad \Rightarrow \quad V_{aa} = \underbrace{\langle\psi_{lm}|\vec{r}|\psi_{lm}\rangle}_{\text{odd f'n}} = 0$$

$$P_{a\to b}(t) = \frac{|V_{ab}|^2}{\hbar^2}\frac{\sin^2\left(\frac{\omega-\omega_0}{2}t\right)}{(\omega-\omega_0)^2} = \left(\frac{\vec{\wp}\cdot\vec{E}}{\hbar}\right)^2 \frac{\sin^2\left(\frac{\omega-\omega_0}{2}t\right)}{(\omega-\omega_0)^2} = P_{b\to a}(t)$$

* Continuous spectrum of states: Incoherent Perturbations

$$|V_{ab}|^2 = |-\vec{\wp}\cdot\vec{E}_0|^2 = E_0^2\,\wp^2\cos^2\theta = \frac{2u}{\epsilon_0}\wp^2\cos^2\theta \qquad \left[u = \frac{1}{2}\epsilon_0 E_0^2\right]$$

$$V_{ab} \sim \left| \frac{-\vec{p}\cdot\vec{E}_0}{\hbar} \right|^2 = \frac{E_0^2}{\hbar^2} p^2 \cdot \cos^2\theta = \frac{2u}{\varepsilon_0 \hbar^2} p^2 \cos^2\theta \qquad \left(u = \tfrac{1}{2}\varepsilon_0 E_0^2 \right)$$

for continuous spectrum $u \to \int d\omega\, \rho(\omega)$

$$P_{b\to a} = \frac{2}{\varepsilon_0 \hbar^2} |p|^2 \langle \cos^2\theta \rangle \int_{0\,(-\infty)}^{\infty} d\omega\, \underbrace{\rho(\omega)}_{\approx\, \omega_0} \frac{\sin^2\left((\omega_0-\omega)t/2\right)}{(\omega_0-\omega)^2} \qquad \left[\int_{-\infty}^{\infty} \left(\frac{\sin x}{x}\right)^2 dx = \pi \right]$$

$$\cong \frac{\pi |p|^2}{\varepsilon_0 \hbar^2} \langle \cos^2\theta \rangle \rho(\omega_0)\, t = \frac{\pi |p|^2}{3\varepsilon_0 \hbar^2} \rho(\omega_0)\, t$$

$$\langle \cos^2\theta \rangle = \frac{1}{4\pi} \int_{4\pi} d\Omega \cos^2\theta = \frac{1}{4\pi} \int_{-1}^{1} dx \int_0^{2\pi} d\phi\, x^2 = \frac{1}{2} \left. \frac{x^3}{3} \right|_{-1}^{1} = \frac{1}{3}$$

thus $R_{b\to a} \equiv \dfrac{dP_{b\to a}}{dt} = \dfrac{\pi |p|^2}{3\varepsilon_0 \hbar^2} \rho(\omega_0)$ $\qquad \vec{p} = q \langle \psi_b | \vec{r} | \psi_a \rangle$

"dipole transition operator"

(historically labelled B_{ba} or B_{ab})