

* Life time of an excited state

- exponential decay rate: const. decay rate per particle

$$dN_b = -\tau N_b dt \quad N_b(t)/N_b(0) = e^{-\tau t} = e^{-t/\tau}$$

where $\tau = R_{b \rightarrow a} = A$ [decay rate] and $\tau = \frac{1}{\Gamma}$ [lifetime]

- decay modes & branching ratio

$$\tau_{\text{tot}} = \tau_1 + \tau_2 + \tau_3 + \dots \quad \text{transition rates } \tau_i \text{ to different states}$$

* Sources of natural line width (decay rate)

http://quantummechanics.ucsd.edu/ph130a/130_notes/node428.html

A) Breit-Wigner Resonance: line shape

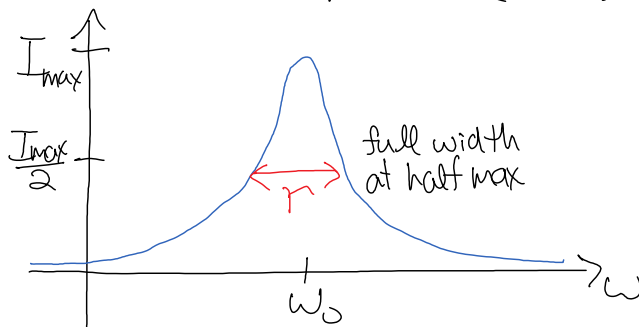
uncertainty principle: $\Delta\omega \approx \frac{1}{\tau} \approx \Gamma$ (line width)

$$P(t) = |\psi(t)|^2 = e^{-\Gamma t} \Rightarrow \psi(t) = \psi_0(x) e^{-iEt/\hbar} e^{-\Gamma t/2}$$

Take the Fourier transform to convert to frequency:

$$\phi(\omega) = \int_0^\infty dt \psi(t) e^{i\omega t} \propto \int_0^\infty e^{i(\omega - \omega_0 + i\Gamma/2)t} dt = \frac{i}{\omega - \omega_0 + i\Gamma/2}$$

$$I(\omega) \propto |\phi(\omega)|^2 = \frac{1}{(\omega - \omega_0)^2 + (\Gamma/2)^2} \quad \text{"Lorentzian function"}$$



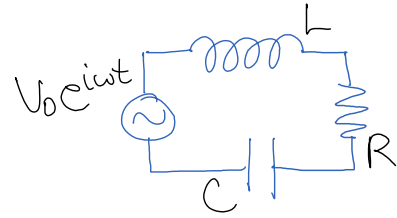
- This line shape is called a "Breit-Wigner Resonance"

- note: $\sigma_\omega = \infty!$ $\Delta E_{FWHM} = \hbar \Gamma$

- Compare with LRC resonance of a tank circuit:

$$I(\omega) = V_0 / Z(\omega) = \frac{V_0}{i\omega L + \frac{1}{i\omega C} + R}$$

$$I_{\max} = \frac{V_0}{R} \quad \text{at } \omega_0^2 = \frac{1}{LC}$$



$$|I(\omega)| = V_0 [(\omega L - \frac{1}{\omega C})^2 + R^2]^{-1/2} = V_0 [(\omega^2 - \omega_0^2)^2 \frac{L^2}{\omega^2} + R^2]^{-1/2}$$

$$\approx V_0 [(\omega - \omega_0)^2 4L^2 + R^2]^{-1/2} = V_0 / R [1 + (\omega - \omega_0)^2 \cdot (\frac{2L}{R})^2]^{-1/2}$$

$$\approx \frac{V_0 / R}{1 + \frac{1}{2}(\omega - \omega_0)^2 \cdot \frac{4L^2}{R^2}} = \boxed{\frac{V_0 R / 2L^2}{(\omega - \omega_0)^2 + (\Gamma/2)^2} \quad \begin{matrix} \omega_0^2 = 1/LC \\ \Gamma = 1/\sqrt{2} R/L \end{matrix}}$$

B) **Collision Broadening**: collisional relaxation (decay)

$$\tau_c = \frac{\# \text{ collisional decays}}{\text{time}} = \frac{\#}{\text{volume}} \cdot \frac{\text{length}}{\text{time}} \cdot \frac{\text{cross-sectional area}}{\text{area}} = n v \sigma = n \sqrt{\frac{3kT}{m}} \sigma$$

$$\text{where } E_{th} = \frac{1}{2} m \bar{v}_{th}^2 = \frac{3}{2} kT$$

C) **Doppler Broadening**: classical frequency shift

$$\frac{\Delta \omega}{\omega} = \frac{v_{th}}{c} = \frac{\sqrt{E_{th}}}{c} = \sqrt{\frac{kT}{mc^2}} = \sqrt{\frac{E_{th}}{E_0}}, \text{ significant if } n c \sigma \ll 1$$

D) **Recoil Line Shift**: conservation of momentum $\rightarrow$ recoil atom

$$|\vec{p}_{\text{atom}} = -\vec{p}_\gamma| = E/c \quad \text{so} \quad \frac{\Delta E}{E} = \frac{p^2/2m}{E} = \frac{(E/c)^2/2m}{E} = \frac{E}{2mc^2} = \frac{E}{2E_0}$$

This is a line shift, not broadening, since recoil is in the opposite direction as photon emission.

It prevents the photon from being reabsorbed.

It is much more important in nuclear physics.