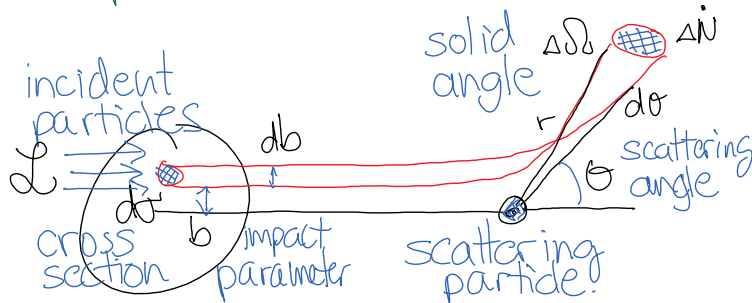


* Classical scattering cross sections §11.1.1

"Differential cross sections $d\sigma \rightarrow \frac{d\sigma}{d\Omega}$ are the meeting point between experimentalists and theorists."



$$\Delta \dot{N} = \underbrace{\mathcal{L}}_{\text{beam}} \frac{d\sigma}{d\Omega} \cdot \underbrace{\Delta\Omega}_{\text{detector}} \quad [\text{expt.}]$$

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{2\pi} \cdot \underbrace{b}_{\text{physics}} \cdot \frac{db}{d\theta} \quad [\text{theory}]$$

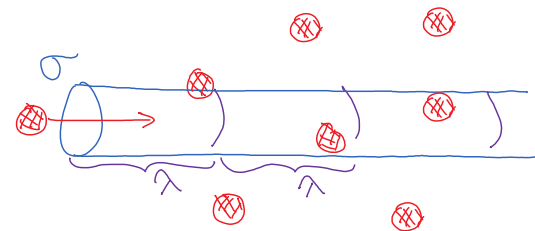
$$\mathcal{L} = \underbrace{\frac{\# \text{projectiles}}{\text{time}}}_{I_{\text{beam}}} \cdot \underbrace{\frac{\# \text{scatters}}{\text{area}}}_{I_{\text{target}}} = \underbrace{\# \text{projectiles}}_N \cdot \underbrace{\frac{\# \text{scatters}}{\text{volume}}}_n \cdot \underbrace{\text{relative velocity}}_{v_{\text{rel}}} \quad [\text{luminosity}]$$

$$\Delta\Omega = \text{detector area} / r^2 = \int_{\text{det}} \underbrace{\sin\theta d\theta d\phi}_{d\Omega} \leq 4\pi \quad [\text{solid angle}]$$

$$\underbrace{\sigma}_{\text{total cross section}} = \frac{dN}{\mathcal{L}} = \int d\sigma = \int \frac{d\sigma}{d\Omega} d\Omega \quad \text{where} \quad \underbrace{\frac{d\sigma}{d\Omega}}_{\text{differential cross section}} = \frac{dN}{\mathcal{L} \Delta\Omega}$$

$$\underbrace{\dot{N}/V}_{\text{reaction rate}} = \frac{\# \text{interactions}}{\text{time} \cdot \text{volume}} = \underbrace{n^2}_{\text{density}} \underbrace{\langle \sigma v_{\text{rel}} \rangle}_{\text{interaction volume rate}} = \left(\frac{\# \text{atoms}}{\text{volume}} \right)^2 \left(\text{area} \cdot \frac{\text{volume}}{\text{time}} \right)^2$$

$$\underbrace{n\lambda\sigma}_{\text{mean free path}} \equiv 1 = \frac{\# \text{atoms}}{\text{volume}} \cdot \text{length per interaction} \cdot \text{area of interaction}$$

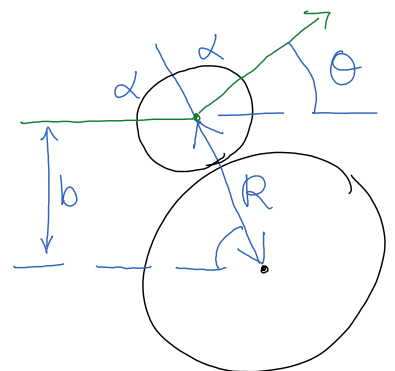


$$N = N_0 e^{-n\lambda\sigma} = N_0 e^{-x/\lambda} \quad [\text{attenuation}]$$

* Example: hard sphere scattering

$$b = R \sin\alpha = R \sin\left(\frac{\pi - \theta}{2}\right) = R \cos\left(\frac{\theta}{2}\right)$$

$$\frac{d\sigma}{d\Omega} = \left| \frac{b db}{\sin\theta d\theta} \right| = \left| \frac{R \cos(\frac{\theta}{2}) \cdot \frac{1}{2} R \sin(\frac{\theta}{2}) d\theta}{2 \cdot 2 \sin(\frac{\theta}{2}) \cos(\frac{\theta}{2}) d\theta} \right| = \frac{1}{4} R^2$$



$$\frac{d\sigma}{d\Omega} = \left| \frac{v_{out}}{v_{in}} \right| = \left| \frac{K \cos(\theta/2) \cdot \bar{K} \sin(\theta/2) d\Omega}{2 \cdot 2 \sin(\theta/2) \cos(\theta/2) d\Omega} \right| = \frac{1}{4} R^2$$

$$\sigma = \int d\sigma = \int_{4\pi} \frac{1}{4} R^2 d\Omega = \pi R^2 \quad \text{geometric interaction area! "short range force"}$$

Note: Coulomb interaction is long-range: $\sigma \rightarrow \infty$!

* Quantum mechanical scattering cross section §11.1.2

- Exactly the same concept, but in terms of probability amplitude, using probability currents instead of fluxes
- wave mechanics obscures plain geometric interpretation
- must be defined by $\frac{d\sigma}{d\Omega} \equiv \frac{\Delta N}{\Delta \Omega}$ instead of $\sigma(b)$
- exact same formalism as 1-d transmission/reflection: solve Schrödinger equation for asymptotic **outgoing wave** $\left[\frac{e^{ikr}}{r} \right]$ with **incoming wave** $[e^{ikz}]$ as external boundary condition

$$\psi(r, \theta) \approx A \left\{ \underbrace{e^{ikz}}_{\text{incoming}} + \underbrace{f(\theta)}_{\text{scattering amplitude}} \underbrace{\frac{e^{ikr}}{r}}_{\text{outgoing}} \right\} \quad \begin{matrix} p = \hbar k \\ E = \frac{\hbar^2 k^2}{2m} \end{matrix} \quad \text{[spherically symmetric scattering potential]}$$

$$\frac{d\sigma}{d\Omega} = \frac{dP_{\text{scat}}/d\Omega}{dP_{\text{inc}}/da} = \frac{|\psi_{\text{scat}}|^2 dV/d\Omega}{|\psi_{\text{inc}}|^2 dV/da} = \frac{|A f(\theta)/r|^2 r^2 (v dt) d\Omega/da}{|A|^2 (v dt) d\sigma/d\sigma} = |f(\theta)|^2$$

- scattering amplitude $f(180^\circ) \approx \frac{B}{A}$ [reflection] $f(0^\circ) \approx \frac{F}{A}$ [transmission] plus all the angles in between!
- generalization: S-matrix contains transition amplitudes from one asymptotic quantum state $[nlm]$ to another $[n'l'm']$
- in order to solve for $f(\theta)$, must write e^{ikz} and $\frac{e^{ikr}}{r}$ in the same basis $\rightarrow$ spherical waves: $j_\ell(kr) Y_{\ell m}(\Omega)$ components of $f(\theta)$ in this basis are "partial wave amplitudes" a_ℓ

in the same basis $\rightarrow$ spherical waves: $j_l(kr) Y_{lm}(\Omega)$
components of $f(\theta)$ in this basis are "partial wave amplitudes" a_l
and can be parametrized as "phase shifts" δ_l
at low energy, where $kl \ll 1$ so $l=0$ dominates

- alternatively, at higher energy, $E \gg V$, we can perturbatively expand Schrödinger's equation, converting it to integral form using Green's functions. This expansion is particularly simple in the Born Approximation, and each term has a corresponding Feynman Diagram.