University of Kentucky, Physics 521
Homework #15, Rev. A, due Tuesday, 2017-02-21

0. Griffiths [2ed] Ch. 4 #34, #36; Ch. 5 #7, #15, #17, #35, #36.

1. Davilov Parameters [Danilov, Phys. Lett. **35B**, 579 (1971)] The parity violating elastic nucleon scattering amplitudes can be classified using symmetry and conservation principles studied in class. The neutron (n) and proton (p) are two states of a single particle, the *nucleon* (N), just as their constituent up (u) and down (d) particles are both quarks (q). In analogy with the two-state system spin, we assign the nucleon *isospin* $I = \frac{1}{2}$. The proton ($I_3 = \frac{1}{2}$) and neutron ($I_3 = -\frac{1}{2}$) are represented by two-component *isospinors* $\Upsilon = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, respectively, with Pauli matrix operators writen $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ as opposed to $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. Addition of isospin is just like spin: the two-particle system form an $I = 0$ singlet (for example, the ω meson) and an $I = 1$ triplet (the ρ^+, ρ^0, ρ^- mesons).

a) For now, neglect the spatial part $\phi(\boldsymbol{r})$ of the nucleon wavefunction $\psi(\boldsymbol{r}, \boldsymbol{S}, \boldsymbol{I}) = \phi(\boldsymbol{r})\chi_{m_S}\Upsilon_{m_I}$. Write down the spin-isospinor $\chi\Upsilon$ for the four different spin and isospin combinations of N. Denote the two spin states by $\uparrow$ and $\downarrow$, and the isospin up and down states by p and n, respectively.

b) Construct all 10 symmetric and 6 antisymmetric NN wavefunctions by [anti]symmetrizing the 16 combinations of two single-particle spin-isospinors. Show that each of these states can be written as a linear combination of states with definite total spin and isospin, ie., the product of a spin singlet or triplet multiplied by an isospin singlet or triplet). What is the particle exchange symmetry of each? [**bonus**: How many completely symmetric and antisymmetric states are there in the NNN system? In the three-quark system, they are p, n, $\Delta^{-,0,+,++}$.]

c) Transform the two-particle spatial wave function $\phi_1(\boldsymbol{r}_1)\phi_2(\boldsymbol{r}_2)$ into CM-relative coordinates $\phi_{\boldsymbol{R}}(\boldsymbol{R})\phi_{\boldsymbol{r}}(\boldsymbol{r})$ about the center of mass and reduced mass. Show that $P_{12}\phi = P_{\boldsymbol{r}}\phi$, where P_{12} is the normal particle exchange operator and $P_{\boldsymbol{r}}$ is parity (spatial inversion) acting on the relative coordinate: $\boldsymbol{r} \to -\boldsymbol{r}$. As free particles, the two spatial wavefunctions (either particle $\boldsymbol{r}_1, \boldsymbol{r}_2$ or $\boldsymbol{R}, \boldsymbol{r}$) have definite orbital angular momentum with quantum numbers ℓ, m_ℓ. Show that the relative angular wave function $Y_{\ell m}(\hat{\boldsymbol{r}})$ is even[odd] under $P_{\boldsymbol{r}}$ for even[odd] ℓ, so that the simplest parity violating amplitudes are S-P ($\ell = 0 \to 1$) transitions.

d) The total wave function $\psi = \phi\chi\Upsilon$ must be antisymmetric under particle exchange P_{12} by the Pauli exclusion principle. Mix even/odd combinations of $Y_{\ell m}(\hat{\boldsymbol{r}})$ from part c) and $\chi\Upsilon$ from part b) to construct all S and P states. Use linear combinations of these to create eigenfunctions of total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$. Denote each state in spectroscopic notation ${}^{2S+1}L_J(I)$, where S,L,J, and $[I]$ are the total [iso]angular momentum quantum numbers of the combined NN system. Use conservation of total angular momentum $\boldsymbol{J}$, to show there are only five allowed S-P transition amplitudes: $\lambda_s(\Delta I = 0, 1, 2)$, $\lambda_t(I = 0)$, and $\rho_t(I = 0 \to 1)$, as illustrated below.

$$\begin{tikzcd}
{}^3P_0(I=1) \arrow[dr, leftrightarrow, "\lambda_s"] & & {}^1P_1(I=0) \arrow[dr, leftrightarrow, "\lambda_t"'] & & {}^3P_1(I=1) \arrow[dl, leftrightarrow, "\rho_t"] \\
& {}^1S_0(I=1) & & {}^3S_1(I=0) &
\end{tikzcd}$$