L70-Zeeman effect

Monday, March 21, 2016 09:37

* review: structure of H-spectrum: dipole interactions.
  perturbative expansion on $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \sim 1/137$

$E^0 \sim \alpha^2 mc^2$ Bohr levels $\longrightarrow E_n^0 = -E_1^0/n^2$

$E^1_{fs} \sim \alpha^4 mc^2$ fine structure $\longrightarrow E_n^1 = \frac{-E_1^0}{n^2}\left[\frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - 3/4\right)\right]$

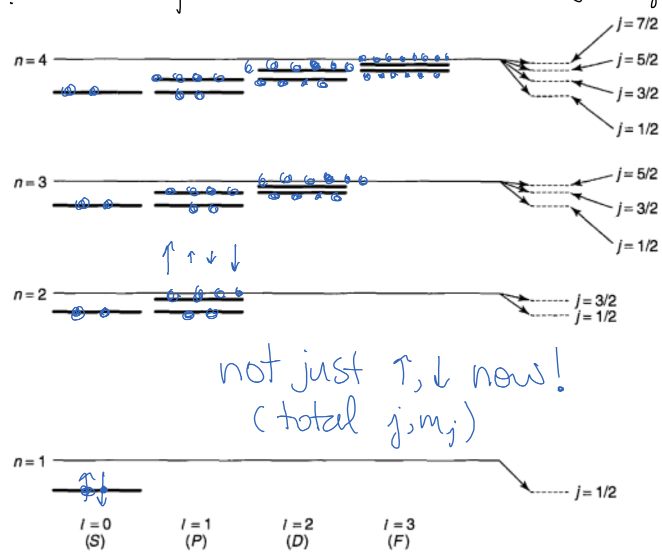
$E^1_{LS} \sim \alpha^5 mc^2$ Lamb shift

$E^1_{hfs} \sim \alpha^4 \frac{m_e}{m_p} mc^2$ hyperfine struct.

• New quantum numbers: j, m_j

$\vec{\mu}\cdot\vec{B}$ interaction μ_e with
• internal magnetic field
  proton orbit – fine structure
  proton spin – hyperfine struct.
• external magnetic field
  Zeeman effect (tunable)

$\vec{p}\cdot\vec{E}$ with external electric field
  Stark effect (also tunable)



* in an external magnetic field, electron energy has an additional perturbation:

$$H_Z' = -(\mu_\ell + \mu_s)\cdot\vec{B}_{ext}$$
$$= \frac{e}{2m}(\vec{L} + 2\vec{S})\cdot\vec{B}_{ext}$$
$$= (g_\ell m_\ell + g_s m_s)\mu_B B_{ext}$$

$\vec{\mu}_\ell = -g_\ell \cdot \frac{e\hbar}{2m} \cdot \vec{L}/\hbar$ $\quad g_\ell = 1$ } Landé

$\vec{\mu}_s = -g_s \cdot \mu_B \cdot \vec{S}/\hbar$ $\quad g_s = 2$ } factors

$\mu_B = 57.88\ \mu eV/T$ Bohr magneton

$\gamma_{\ell,s} = g_{\ell,s}\cdot e/2m$ gyromagnetic ratio.

* perturbation theory depends on which "good" quantum numbers break the degeneracy of the Bohr energy levels.
  a) if $B_{int} \gg B_{ext}$ then j, m_j good quantum numbers.
  b) ... $B_{int} \ll B_{ext}$ ... m_ℓ, m_s - - - -
  c) $B_{int} \approx B_{ext}$ must diagonalize complete perturbation.

A) weak-field Zeeman effect:

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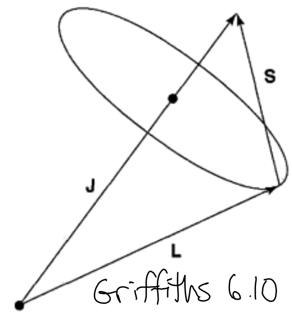
quantum #'s: n, ℓ, j, m_j but not m_ℓ, m_s



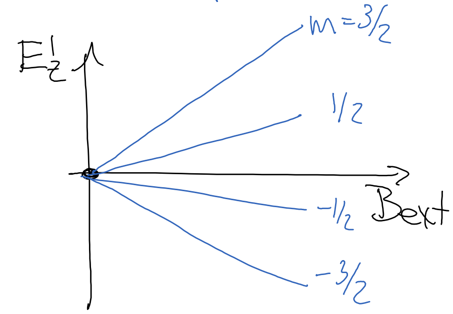
Griffiths 6.10

- we must find the time-ave of m_ℓ, m_s in $\mathcal{H}_Z'$
- $\vec{L}, \vec{S}$ orbit around $\vec{J}$, find their projection!

$$\langle \vec{L} + 2\vec{S} = \vec{J} + \vec{S} \rangle = \left\langle \vec{J}\left(1 + \frac{\vec{J}\cdot\vec{S}}{J^2}\right)\right\rangle = \underbrace{\left(1 + \frac{j(j+1) - \ell(\ell+1) + s(s+1)}{2\, j(j+1)}\right)}_{g_J \text{ Landé factor}} \vec{J}$$

where $\left(\vec{L} = \vec{J} - \vec{S}\right)^2 = J^2 + S^2 - 2\vec{J}\cdot\vec{S}$

- thus $E_Z^1 = g_J \mu_B m_j B_{ext}$



m = "magnetic" quantum number.

B-field breaks the m-degeneracy.

B) Strong-field Zeeman effect $\quad E_Z' = \frac{e}{2m} B_{ext} \cdot (\vec{L} + 2\vec{S})$

quantum #'s: n, ℓ, m_ℓ, m_s ($\mathcal{H}_Z'$ breaks the degeneracy of n)

- E_{fs} is a perturbation to: $E_{n\ell m_\ell m_s} = \frac{-E_1}{n^2} + (m_\ell + 2m_s)\mu_B B_{ext}$

$$E_r^1 = \frac{-(E_n)^2}{2mc^2}\left[\frac{4n}{\ell + 1/2} - 3\right] \text{ (same)} \qquad \mathcal{H}_{SO}' = \frac{e^2}{8\pi\varepsilon_0} \frac{1}{m^2c^2r^3} \underbrace{S\cdot L}_{\hbar^2 m_\ell m_s}$$

- instead of $\vec{J}^2 = (\vec{L} + \vec{S})^2 = L^2 + 2\vec{L}\cdot\vec{S} + S^2 \quad \rightarrow \quad L\cdot S = \frac{1}{2}(J^2 - L^2 - S^2)$

use $\langle \vec{S}\cdot\vec{L} \rangle = \langle \cancel{S_x L_x} + \cancel{S_y L_y} + S_z L_z \rangle = S_z L_z = \hbar^2 m_\ell m_s$

$$E_{fs}^1 = \frac{E_1}{n^3}\alpha^2 \left\{\frac{3}{4n} - \frac{\ell(\ell+1) - m_\ell m_s}{\ell(\ell + 1/2)(\ell+1)}\right\}$$

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* defer "Intermediate Field Zeeman Effect" to "Hyperfine Zeeman splitting."