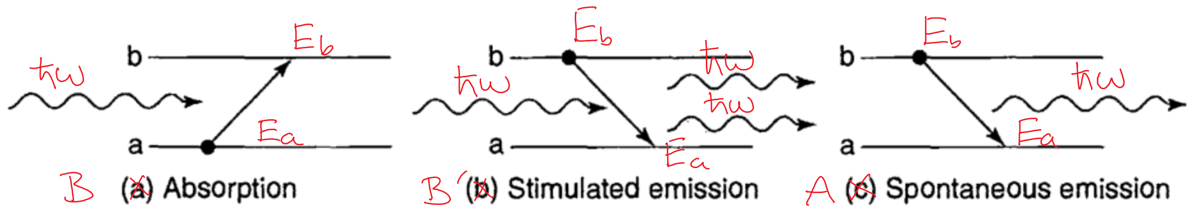
L77-Spontaneous Emission

Wednesday, April 6, 2016 15:34

* review: processes of radiation:



B) absorption $E_a + \hbar\omega_0 \to E_b$ strongly peaked at $\omega \approx \omega_0$ (photon)

B′) stimulated emission $E_b \to E_a + \hbar\omega$ same probability!

- Recall "Rabi flopping" between spinstates in osc. field exact solution, not just perturbative.

- Nonclassical: predicted by Einstein in 1917 by comparing emission/absorption of black body radiation (Planck, 1900) Predicted ~ 10 yrs before Heisenberg/Schrödinger eq! (1926)

- Laser (Light Amplification by Stimulated Emission of Radiation) uses this process to amplify coherent radiation, requires:

  i) population inversion: so that $b \to a$ dominates $a \to b$ must "pump" atoms from $a \to b$ state

  ii) cavity: multiple passes of photon enhances stimulated emission and sharpens wavelength of laser.

A) spontaneous emission

- classical process would be forbidden quantum-mechanically without a perturbation.

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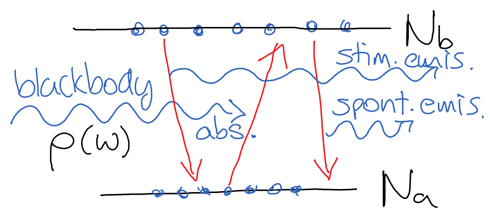
- in QED, the vacuum (ground state) includes "zero-point radiation", like $E_0 = \frac{1}{2}\hbar\omega$ ground state of harmonic oscillator.
  problem # 9.9

- zero pt. radiation responsible for "Casimir force" between parallel plates

- thus Quantum Mechanics: ALL radiation is stimulated
  vs. Classical Mechanics: ALL radiation is spontaneous.

- also "thermally stimulated emission", stimulated by black body radiation, at low frequencies $\omega \ll$ THz (300K)
  problem # 9.8

* Einstein's A, B coefficients



recall: $R_{b\to a} \equiv \frac{dP_{b\to a}}{dt} = \underbrace{\frac{\pi |\wp|^2}{3\epsilon_0 \hbar^2}}_{B} \rho(\omega_0)$

$B_{ab}\,\rho(\omega_0)\,N_a$ = rate of stimulated absorption

$B_{ba}\,\rho(\omega_0)\,N_b$ = rate of stimulated emission

$A_{ba}\,N_b$ = rate of spontaneous emission

detailed balance: assuming thermal equilibrium:

$$\dot{N}_b = -N_b A - N_b B_{ba}\,\rho(\omega_0) + N_a B_{ab}\,\rho(\omega_0) = 0$$

$$\rho(\omega_0) = \frac{A}{\frac{N_a}{N_b} B_{ab} - B_{ba}} = \frac{A}{e^{\frac{\hbar\omega_0}{kT}} B_{ab} - B_{ba}} = \frac{\hbar}{\pi^2 c^3}\,\frac{\omega_0^3}{e^{\frac{\hbar\omega_0}{kT}} - 1}$$

where $N_a \sim e^{-E_a/kT}$, $N_b \sim e^{-E_b/kT}$, using Blackbody distribution.

thus $B_{ab} = B_{ba} = \frac{\pi |\wp|^2}{3\epsilon_0 \hbar^2}$, $\quad A = \frac{\hbar\omega^3}{\pi^2 c^3}\cdot B = \frac{\omega_0^3}{3\pi\epsilon_0 \hbar c^3}$