University of Kentucky, Physics 521
Homework #17, Rev. B, due Tuesday, 2018-04-03

0. Griffiths [2ed] Ch. 7 #1, #6, #10, #14, #16, #19.

2. Hyperfine Zeeman splitting in deuterium is similar to hydrogen, except that the nucleus is a spin $I = 1$ deuteron instead of a spin $I = \frac{1}{2}$ proton. Thus the 1s ground state has 6-fold degeneracy, including hyperfine structure. Since the deuteron is a loosely bound proton–neutron isosinglet ($T = 0$, note the new notation) dominated by $L = 0$, its magnetic moment is approximately the sum of its constituents $\mu_d = g_I \mu_N = 0.857\ \mu_N \approx \mu_p + \mu_n = 2.79\ \mu_N - 1.91\ \mu_N$.

a) Using the Pauli exclusion principle, show that the deuteron must be a spin triplet $(I = 1)$.

b) Show that after integrating over the spatial wave function, the hyperfine perturbation to the Hamiltonian is $\mathcal{H}'_{\rm hf} = b\vec{\boldsymbol{I}} \cdot \vec{\boldsymbol{S}}$ and calculate b for deuterium. Show that the perturbation for the Zeeman effect is $\mathcal{H}'_{\rm Z} = (g_S \mu_B \vec{\boldsymbol{S}}/\hbar + g_I \mu_N \vec{\boldsymbol{I}}/\hbar) \cdot \vec{\boldsymbol{B}}_{ext}$, ignoring $\vec{\boldsymbol{L}}$. Compare the magnitude of g_S and g_I for the deuterium. Why can we ignore fine structure when considering the degeneracy of the 1s states?

c) Using degenerate perturbation theory, calculate the hyperfine energy shift for deuterium in a weak external magnetic field $B_{ext} \ll B_{int}$. What are the eigenstates and good quantum numbers which break the degeneracy of the Bohr levels? Which quantum numbers are still degenerate? Calculate the Landé factor g_F for the Zeeman shift $(g_F \mu_B \vec{\boldsymbol{F}}/\hbar) \cdot \vec{\boldsymbol{B}}_{ext}$. At what field $B_{int} = b\hbar/g_F \mu_B$ are the two perturbations are approximately equal? Plot the Zeeman shift as a function of B_{ext}.

d) In a large external field, $B_{int} \ll B_{ext}$, we must first break the degeneracy according to the Zeeman shift, which dominates hyperfine structure. Calculate the perturbed energies as a function of B_{ext}. What are the good quantum numbers and corresponding eigenstates? What degeneracies remain? Use these states to calculate the hyperfine energy shift (constant for B_{ext}).

e) In intermediate fields where both perturbations are of the same order, we must diagonalize $\mathcal{H}'_{\rm hf} + \mathcal{H}'_{\rm Z}$ together. Calculate this 6×6 matrix in the $nlm_l m_s m_I$ basis and diagonalize it to obtain the exact energy shifts. Plot the energies as a function of B_{ext}. Show that the small- and large-field limits match (b) and (c) respectively. Repeat the calculation and diagonalization of matrix elements in the $nljFM_F$ basis to show that the result is independent of basis.