University of Kentucky, Physics 521
Homework #18, Rev. A, due Wednesday, 2017-04-11

0. Griffiths [2ed] Ch. 9 #1, #6, #7, #8, #9.

1. The **Lyman-alpha** (L_α) line ($n = 2 \to 1$) is the lowest energy transition to the atomic hydrogen ground state. It is an important wavelength for spectroscopy and laser-cooling antihydrogen.

a) Calculate the energy and wavelength of this transition.

b) Which $n = 2 \to 1$ substates satisfy the electric dipole selection rules?

c) Calculate the lifetime and line width of this transition.

d) Calculate the effects of collisional broadening [use $\sigma_{col} = \pi(\langle r^2\rangle_{100} + \langle r^2\rangle_{211})$, see Griffiths problem #4.13], Doppler broadening, and recoil line shift at STP (20°C, 1 atm), as fractions of the natural wavelength and line width. [Note, however, that UV light attenuates rapidy in air!]

2. Lasers require a *population inversion* of occupation numbers $N_b > N_a$ to achieve a higher rate of *stimulated emission* than of absorption of photons in the transition $E_b \to E_a$. An *optical cavity* with mirrors at each end reflects the laser beam back and forth to create a standing wave, enhancing the photon density and narrowing the line width. A partial silvered mirror on one end allows a fraction of the photons to exit as the laser beam, contributing to attenuation of photons in the cavity. Lasing occurs if there is a net gain of photons in the laser due to stimulated emission.

a) Integrate the spectral density of the Breit-Wigner resonance

$$\rho(\omega) = \frac{\rho(\omega_0)(\Gamma/2)^2}{(\omega - \omega_0)^2 + (\Gamma/2)^2}$$

to obtain the energy density $u = \int_0^\infty \rho(\omega)d\omega \approx \int_{-\infty}^\infty \rho(\omega)d\omega$ in the cavity. Convert u to N_γ, the number of photons in the beam, which occupies a volume V in the cavity.

b) Show that $\dot{N}_\gamma$, the rate of change of the number of photons in the cavity [due to stimulated emission, absorption, and attenuation] is positive if the population inversion is greater than the *critical density*

$$\Delta n_c \equiv \frac{N_b - N_a}{V} = \frac{\omega^2\ \Gamma\tau_s}{2\pi c^3 \tau_\gamma} = \frac{2\pi\Gamma\tau_s}{\lambda^2 c \tau_\gamma},$$

neglecting the degeneracy of states, where τ_s is the lifetime of spontaneous emission, and τ_γ is the characteristic lifetime of photons in the laser cavity.

c) Calculate the population inversion density needed for a Ruby laser [the first pulsed laser] and a He-Ne laser [the first CW laser], using the following values [Tipler & Llewellyn, 5ed]:

	Ruby laser	He-Ne laser
λ	694.3 nm	632.8 nm
n (refractive index)	1.76	1.00
τ_s	3 ms	100 ns
τ_γ	29 ns	330 ns
$\delta\nu = \Gamma/2\pi$	330 GHz	0.9 GHz.

3. **[bonus]** Design an efficient, economic Lyman-alpha laser.