L50-Introduction and review

Tuesday, January 10, 2017   22:35

* Phy 521 Schedule:

4.3-4	angular momentum & spin	
5	identical particles: bosons & fermions	↓ EXAM 1
6-7	approximation methods	
9	time-dependent Hamiltonians & transitions	
11	scattering	FINAL EXAM ↓
[10,12]	quantum phases, philosophy: group projects	

* Postulates of QM – we spent last semester learning & applying them

1) Hilbert space of states: superposition postulate   * why?
State $|\psi\rangle$ = collection of complex probability amplitudes $\psi(x)$ or c_n (vector components) which linearly combine to form new states
It has an inner product $\langle\psi|\psi\rangle = \int dx|\psi(x)|^2 = \sum_n |c_n|^2$
and is normalizable $\langle\psi|\psi\rangle = 1$ so probabilities add to 100%

2) Hermitian observables: expansion/projection postulate
observable = collection of determinate states & measurements (operator $\hat{Q}$) with real eigenvalues q_n & orthogonal eigenvectors $|\phi_n\rangle$ which form a complete set of basis vectors: $|\psi\rangle = \sum_n a_n|\phi_n\rangle$
- $|a_n|^2$ = probability of measuring q_n
- $|\psi\rangle \to |\phi_n\rangle$ after measuring q_n * why operators?

3) Hamiltonian: evolution postulate
states evolve in time according to Schrödinger Eq: $\hat{H}|\Psi\rangle = \hat{E}|\Psi\rangle$
- eigenstates of $\hat{H}$ are stationary states: $\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle$
- evolution of mixed states: $|\Psi(x,t)\rangle = \sum_n c_n|\psi_n\rangle e^{-iE_nt/\hbar}$

4) Heisenberg: uncertainty postulate
- canonical commutation relation $[\hat{x},\hat{p}] = i\hbar$ for conjugate observables
- position and momentum are complementary: $\Delta x\,\Delta p \geq \hbar/2$

- momentum operator $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ and eigenstates $\psi_p(x) = e^{ipx/\hbar}$

5) Pauli: exclusion postulate
   - identical particle exchange symmetry $\psi(x_1, x_2) = \pm\psi(x_2, x_1)$
   - only one fermion can occupy each state

     +: bosons $s = 0, 1, 2, \ldots$
     −: fermions $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots$

   (will discuss this semester: Ch. 5)

* Solution of 3-d free particle TDSE $\hat{H}\Psi = \hat{E}\Psi$ $E = \hbar\omega = \frac{\hbar^2k^2}{2m}$

$$\hat{H}\Psi = \left\{\frac{\hat{p}^2}{2m} + \hat{V}\right\}\Psi = \left[\frac{-\hbar^2}{2m}\nabla^2 + \cancel{V(r)}^{\,0}\right]\Psi(\vec{r},t) = i\hbar\partial_t\Psi(\vec{r},t) = \hat{E}\Psi$$

$$\left[\nabla^2_{sph} = \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}}_{-k_r^2} + \frac{1}{r^2}\left[\underbrace{\frac{1}{S_\theta}\frac{\partial}{\partial\theta}S_\theta\frac{\partial}{\partial\theta}}_{-\ell(\ell+1)} + \frac{1}{S_\theta^2}\underbrace{\frac{\partial^2}{\partial\phi^2}}_{-m^2}\right] + k^2\right]\left\{\psi = j_\ell(k_r r)\underbrace{P_\ell^{|m|}(c_\theta)e^{im\phi}}_{\propto Y_{\ell m}(\theta,\phi)}e^{-i\omega t}\right] = 0$$

Angular momentum operator: $L^2 = -\hbar^2\left[\frac{1}{S_\theta}\frac{\partial}{\partial\theta}S_\theta\frac{\partial}{\partial\theta} + \frac{1}{S_\theta^2}\frac{\partial^2}{\partial\phi^2}\right]$ $L_z = i\hbar\frac{\partial}{\partial\phi}$

space quantization: $L^2\,Y_{\ell m} = \hbar^2\ell(\ell+1)\,Y_{\ell m}$ $L_z Y_{\ell m} = \hbar m\,Y_{\ell m}$

Section 4.3 – we will repeat this using operator methods.

* summary of Sturm-Liouville systems: Hermitian operators & orthogonal functions:

$L|n\rangle = |n\rangle\lambda$ $\frac{1}{\omega}\left(\frac{d}{dx}p\frac{d}{dx} - q\right]f_n(x) = \lambda f_n(x)$ $\langle n|n'\rangle = \int_a^b w\,dx\, f_n^*(x)\, f_{n'}(x) = \delta_{nn'}h_n$

	$f_n(x)$	index	a	b	wdx	$+p$ / $-p$	$+q$ / $-q$	$-\lambda$ / λ	h_n	wave type
Angular i)	$e^{im\phi}$	$m\in\mathbb{Z}$	$0<\phi<2\pi$		$d\phi$	1	0	m^2	2π	(cyl. harmonics)
ii)	$P_\ell^{\|m\|}(x)$	$\ell = 0,1,2..$	$-1<x<1$		dx	$1-x^2$	$\frac{m^2}{1-x^2}$	$\ell(\ell+1)$	$\frac{2}{2\ell+1}\frac{(\ell+\|m\|)!}{(\ell-\|m\|)!}$	(sph. harmonics,
	(c_θ)	"	$0<\theta<\pi$		$S_\theta d\theta$	S_θ	m^2/S_θ	"	"	polar coords)
Radial (free) iii)	$\sin(k_n x)$	$k_n = \frac{n\pi}{b}$	$0<x<b$		dx	1	0	k_n^2	$\frac{b}{2}$	(linear wave)
iv)	$J_m(k_n\rho)$	$k_n = \frac{\beta_{nm}}{b}$	$0<\rho<b$		$\rho d\rho$	ρ	$\frac{m^2}{\rho}$	k_n^2	$\frac{b^2}{2}J^2_{m\pm1}(\beta_{nm})$	(circular wave)

iv)	$J_m(k_n\rho)$ $k_n=\frac{\beta_{nm}}{b}$	$0<\rho<b$	$\rho\,d\rho$	ρ	$\frac{m^2}{\rho}$	k_n^2	$\frac{b^2}{2}J_{m\pm1}^2(\beta_{nm})$	(circular wave)
v)	$j_\ell(k_n r)$ $k_n=\frac{\beta_{n\ell}}{b}$	$0<r<b$	r^2dr	r^2	$\frac{\ell(\ell+1)}{r^2}$	k_n^2	$\frac{b^2}{2}j_{\ell\pm1}^2(\beta_{n\ell})$	(spherical wave)
Radial (potentials)								
vi)	$Ai(x+x_n)$ $n=1,2,3\ldots$	$0<x<\infty$	dx	1	x	x_0	$Ai'(x_n)^2$	(linear potential)
vii)	$H_n(x)$ $n=0,1,2\ldots$	$-\infty<x<\infty$	$e^{-x^2}dx$	e^{-x^2}	0	$2n$	$\sqrt{\pi}\,2^n n!$	(1-d oscillator)
viii)	$L_n^{(\alpha)}(x)$ $n=0,1,2\ldots$	$0<x<\infty$	$x^\alpha e^{-x}dx$	$x^{\alpha+1}e^{-x}$	0	n	$\frac{\Gamma(\alpha+1+n)}{n!}$	(Harmonic osc. Coulomb potential)

* Properties of commutators:

i) Multilinear $[\alpha_i A_i, B] = \alpha_i [A_i, B]$

$[A, \beta_i B_i] = \beta_i [A, B_i]$

ii) Antisymmetric $[A,B] = -[B,A]$

iii) Jacobi identity $[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$

"Lie Algebra" same properties as the cross product!

iv) Derivation $[A,BC] = [A,B]C + B[A,C]$

$[AB,C] = [A,C]B + A[B,C]$

like the product rule for derivatives

* review the operator method for the harmonic oscillator.

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x) \qquad [a_-,a_+] = \frac{i}{i\hbar}[x,p] = 1$$

$$\mathcal{H} = \hbar\omega\left(a_-a_+ - \tfrac{1}{2}\right) = \hbar\omega\left(a_+a_- + \tfrac{1}{2}\right)$$

let $\mathcal{H}|n\rangle = E_n|n\rangle$, use $[\mathcal{H},a_\pm] = \pm\hbar\omega a_\pm$

then $\mathcal{H}a_\pm|n\rangle = (a_\pm\mathcal{H} \pm \hbar\omega a_\pm)|n\rangle = (E_n \pm \hbar\omega)\, a_\pm|n\rangle$

and $a_\pm|n\rangle$ is a different eigenstate with higher/lower energy