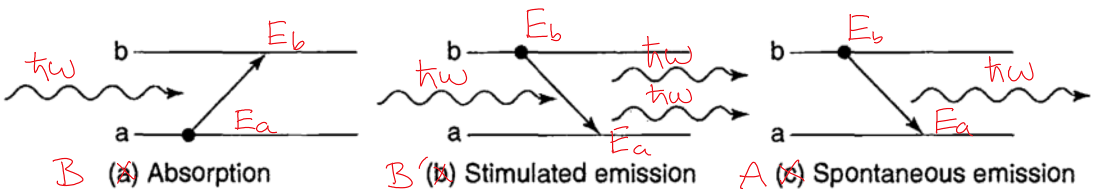
L76-Emission and Absorption of Radiation

Wednesday, April 6, 2016 08:47

* there are three processes involved in atomic transitions:

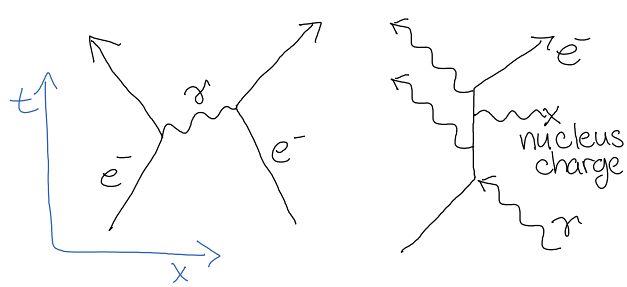


- Today we will use our results in 2-state time-dependent perturbation theory to calculate the transition rates of the first two processes, which are theoretically the same: electronic transition in the presence of an electromagnetic perturbation.

- Next day we will explore spontaneous emission, but we will have to use a statistical argument (Blackbody Radiation)

- Note that the light is only considered as a perturbation of the Hamiltonian in QM. Outside of QM, we rely on conservation of energy and $E = \hbar\omega$ to deduce that a "photon" was emitted or absorbed.

- There is no notion of "photon" in standard QM, also called "first quantization": quantization of energy levels of matter (the electron).
  The photon naturally debuts in the "second quantization" of fields, or radiation, such as E&M (light) waves. This is called "Quantum Field Theory", and involves "Feynman diagrams" with photons, which can be created and destroyed.
  The creation and annihilation of a photon is completely analogous to $a^\dagger$ & a, the creation and annihilation operators of SHO energy quanta.



* Sinusoidal Perturbations: $\mathcal{H}' = V(\vec{r})\cos(\omega t)$

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$\mathcal{H}'_{ab} = V_{ab}\cos\omega t$ where $V_{ab} = \langle\psi_a|V|\psi_b\rangle$ as usual

$$c_b^{(1)} = \frac{1}{i\hbar}\int_0^t dt\, V_{ba}\cos\omega t\, e^{i\omega_0 t} = \frac{V_{ba}}{2i\hbar}\int_0^t dt'\left(e^{i(\omega_0+\omega)t'} + e^{i(\omega_0-\omega)t'}\right)$$

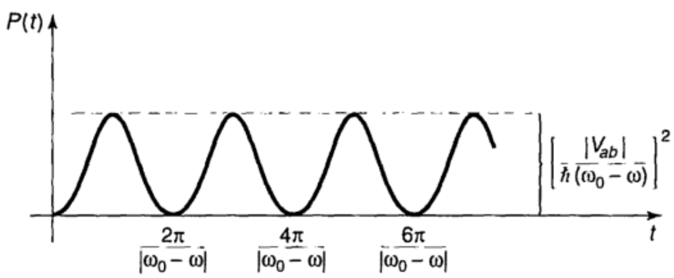
$$= \frac{-V_{ba}}{2\hbar}\left[\cancel{\frac{e^{i(\omega_0+\omega)t}-e^0}{\omega_0+\omega}} + \frac{e^{i(\omega_0-\omega)t}-e^0}{\omega_0-\omega}\right]$$

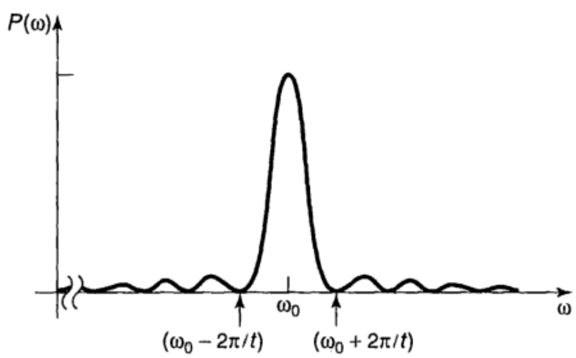
small

$$\cong \frac{V_{ba}}{i\hbar}\,\frac{\sin\left(\frac{\omega_0-\omega}{2}t\right)}{\omega_0-\omega}\, e^{i\frac{\omega-\omega_0}{2}t}$$

$$P_{a\to b}(t) = |c_b^{(1)}|^2 = \frac{|V_{ab}|^2}{\hbar^2}\,\frac{\sin^2\left(\frac{\omega-\omega_0}{2}t\right)}{(\omega-\omega_0)^2} \quad \text{for} \quad |V_{ab}|^2 \ll \hbar^2(\omega-\omega_0)^2$$

* the exact solution is called "Rabi flopping" (HW14)





* Monochromatic EM waves - Electric Dipole Approximation

$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \approx \vec{E}_0\cos\omega t$ if $\vec{k}\cdot\vec{r} \ll 2\pi$ ie. $\frac{2\pi}{k} = \lambda \gg a$

$\mathcal{H}' = -\hat{\vec{p}}\cdot\vec{E} = -q\,\vec{r}\cdot\vec{E}_0\cos\omega t$ so $V(\vec{r}) = -\hat{\vec{p}}\cdot\vec{E}_0$ operator

$\mathcal{H}_{ab} = \langle\psi_a|\mathcal{H}'|\psi_b\rangle = -\vec{p}\cdot\vec{E} = V_{ab}\cos\omega t \qquad V_{ab} = -\vec{p}\cdot\vec{E}_0$

where $\vec{p} = \langle\psi_a|\hat{\vec{p}}|\psi_b\rangle = q\langle\psi_a|\vec{r}|\psi_b\rangle$

$P_{\vec{r}}\,\psi_{lm}(\vec{r}) \equiv \psi_{lm}(-\vec{r}) = \underbrace{(-1)^l\,\psi_{lm}(\vec{r})}_{\text{either even or odd}} \Rightarrow V_{aa} = \underbrace{\langle\psi_{lm}|\vec{r}|\psi_{lm}\rangle}_{\text{odd fn}} = 0$

$$P_{a\to b}(t) = \frac{|V_{ab}|^2}{\hbar^2}\frac{\sin^2\left(\frac{\omega-\omega_0}{2}t\right)}{(\omega-\omega_0)^2} = \left(\frac{\vec{\wp}\cdot\vec{E}}{\hbar}\right)^2 \frac{\sin^2\left(\frac{\omega-\omega_0}{2}t\right)}{(\omega-\omega_0)^2} = P_{b\to a}(t)$$

* Continuous spectrum of states: Incoherent Perturbations

$$V_{ab} \sim \left|\frac{-\vec{\wp}\cdot\vec{E}_0}{\hbar}\right|^2 = \frac{E_0^2}{\hbar^2}\wp^2\cdot\cos^2\theta = \frac{2u}{\varepsilon_0\hbar^2}\wp^2\cos^2\theta \qquad \left(u = \tfrac{1}{2}\varepsilon_0 E_0^2\right)$$

for continuous spectrum $u \to \int d\omega\, \rho(\omega)$

$$P_{b\to a} = \frac{2}{\varepsilon_0\hbar^2}|\wp|^2\langle\cos^2\theta\rangle \int_{0\,(-\infty)}^{\infty} d\omega\, \underbrace{\rho(\omega)}_{\approx\,\omega_0} \frac{\sin^2(\omega_0-\omega)t/2}{(\omega_0-\omega)^2} \qquad \left[\int_{-\infty}^{\infty}\left(\frac{\sin x}{x}\right)^2 dx = \pi\right]$$

$$\cong \frac{\pi|\wp|^2}{\varepsilon_0\hbar^2}\langle\cos^2\theta\rangle\,\rho(\omega_0)\,t = \frac{\pi|\wp|^2}{3\varepsilon_0\hbar^2}\rho(\omega_0)\,t$$

$$\langle\cos^2\theta\rangle = \frac{1}{4\pi}\int_{4\pi} d\Omega\,\cos^2\theta = \frac{1}{4\pi}\int_{-1}^{1}dx\int_0^{2\pi}d\phi\; x^2 = \frac{1}{2}\left.\frac{x^3}{3}\right|_{-1}^{1} = \frac{1}{3}$$

thus $\Gamma \equiv R_{b\to a} \equiv \frac{dP_{b\to a}}{dt} = \frac{\pi|\wp|^2}{3\varepsilon_0\hbar^2}\rho(\omega_0)$ $\qquad \vec{\wp} = q\langle\psi_b|\vec{r}|\psi_a\rangle$

(historically labelled B_{ba} or B_{ab} [Einstein]) $\qquad$ "dipole transition matrix element"