L78-Line width

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* Lifetime of an excited state

- exponential decay rate: const. decay rate per particle

$dN_b = -\Gamma N_b \, dt \qquad N_b(t)/N_b(0) = e^{-\Gamma t} = e^{-t/\tau}$

where $\Gamma = R_{b\to a} = A$ [decay rate] and $\tau = \frac{1}{\Gamma}$ [lifetime]

- decay modes & branching ratio

$\Gamma_{tot} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \ldots$ transition rates Γ_i to differant states

* Sources of natural line width (decay rate)

http://quantummechanics.ucsd.edu/ph130a/130_notes/node428.html

A) Breit-Wigner Resonance: line shape

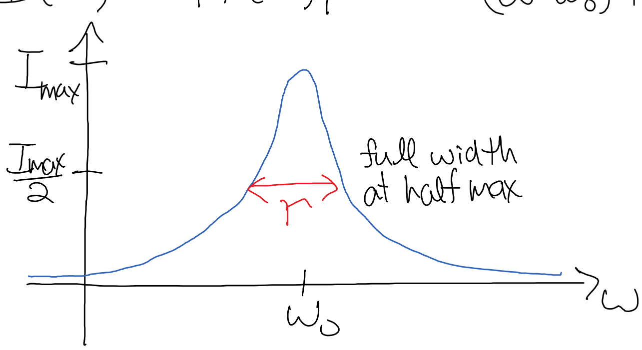
uncertainty principle: $\Delta\omega \approx \frac{1}{\tau} \approx \Gamma$ (line width)

$P(t) = |\psi(t)|^2 = e^{-\Gamma t} \Rightarrow \psi(t) = \psi_0(x)\, e^{-iEt/\hbar}\, e^{-\Gamma t/2}$

Take the Fourier transform to convert to frequency:

$$\phi(\omega) = \int_0^\infty dt\, \psi(t) e^{i\omega t} \propto \int_0^\infty e^{i(\omega-\omega_0+i\Gamma/2)t}\, dt = \frac{i}{\omega-\omega_0+i\Gamma/2}$$

$$I(\omega) \propto |\phi(\omega)|^2 = \frac{1}{(\omega-\omega_0)^2+(\Gamma/2)^2}$$ "Lorentzian function"

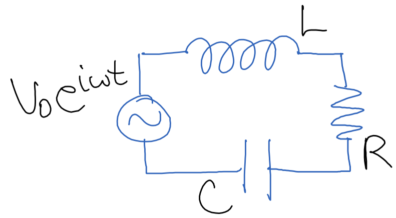


- This line shape is called a "Breit-Wigner Resonance"

- note: $\sigma_\omega = \infty$! $\Delta E_{FWHM} = \hbar\Gamma$

- Compare with LRC resonance of a tank circuit:

$$I(\omega) = V_0/Z(\omega) = \frac{V_0}{i\omega L + \frac{1}{i\omega C} + R}$$



$$I_{max} = \frac{V_0}{R} \quad \text{at } \omega_0^2 = \frac{1}{LC}$$

$$|I(\omega)| = V_0\left[\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2\right]^{-1/2} = V_0\left[(\omega^2-\omega_0^2)^2 \frac{L^2}{\omega^2} + R^2\right]^{-1/2}$$

$$\approx V_0\left[(\omega-\omega_0)^2 4L^2 + R^2\right]^{-1/2} = V_0/R\left[1 + (\omega-\omega_0)^2 \cdot \left(\frac{2L}{R}\right)^2\right]^{-1/2}$$

$$\approx \frac{V_0/R}{1 + \frac{1}{2}(\omega-\omega_0)^2 \cdot \frac{4L^2}{R^2}} = \boxed{\frac{V_0 R/2L^2}{(\omega-\omega_0)^2 + (\Gamma/2)^2} \qquad \begin{aligned}\omega_0^2 &= 1/LC \\ \Gamma &= \tfrac{1}{\sqrt{2}} R/L\end{aligned}}$$

B) Collision Broadening: collisional relaxation (decay)

$$\Gamma_c = \frac{\#\text{collisional decays}}{\text{time}} = \frac{\#}{\text{volume}} \cdot \frac{\text{length}}{\text{time}} \cdot \text{cross-sectional area} = nv\sigma = n\sqrt{\frac{3kT}{m}}\,\sigma$$

where $E_{th} = \frac{1}{2} m v_{th}^2 = \frac{3}{2} kT$

C) Doppler Broadening: classical frequency shift

$$\frac{\Delta\omega}{\omega} = \frac{v_{\parallel}}{c} = \frac{\sqrt{kT/m}}{c} = \sqrt{\frac{kT}{mc^2}} = \sqrt{\frac{E_{th}}{E_0}},$$ significant if $nc\sigma \ll 1$

D) Recoil Line Shift: conservation of momentum $\rightarrow$ recoil atom

$$|\vec{p}_{atom} = -\vec{p}_\gamma| = E_\gamma/c \quad \text{so} \quad \Delta E_\gamma = \frac{p^2}{2m} = \frac{(E_\gamma/c)^2}{2m} = \frac{E_\gamma^2}{2mc^2} = \frac{E_\gamma^2}{2E_0}$$

This is a line shift, not broadening, since recoil is in the opposite direction as photon emission.

It prevents the photon from being readsorbed.

It is much more important in nuclear physics.