L79-Selection Rules

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* Example: Harmonic Oscillator Radiation

(charge q attached to spring $k = m\omega_0^2$ in x-axis)

$$\vec{p} = q\langle n|\hat{x}|n'\rangle \hat{x} = q\sqrt{\frac{\hbar}{2m\omega}}(a_- + a_+) = q\sqrt{\frac{\hbar}{2m\omega}}(\underbrace{\sqrt{n'}\,\delta_{n,n'-1}}_{\text{emission}} + \underbrace{\sqrt{n}\,\delta_{n',n-1}}_{\text{absorption}})$$

using $a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega\hat{x} \mp i\hat{p})$; $a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$ $a_-|n\rangle = \sqrt{n}\,|n-1\rangle$

The selection rule is: $\boxed{\Delta n = 1}$ States separated by one photon!

Radiation frequency: $\hbar\omega_0 = \Delta E = \hbar\omega\Delta n' = \hbar\omega$ Classical!

$$A = \frac{\omega_0^3|\wp|^2}{3\pi\varepsilon_0\hbar c^3} = \frac{\omega_0^3}{3\pi\varepsilon_0\hbar c^3}\cdot\frac{q\hbar n}{2m\omega_0} = \frac{nq^2\omega^2}{6\pi\varepsilon_0 mc^3} = \Gamma = \frac{1}{\tau_n}$$

$$P = A\cdot\hbar\omega = \frac{q^2\omega^2}{6\pi\varepsilon_0 mc^3}(E - \underbrace{\tfrac{1}{2}\hbar\omega}_{\text{quantum diff.}}) \quad \text{vs.} \quad \frac{q^2\ddot{x}^2}{6\pi\varepsilon_0 c^3} = \frac{q^2 x_0^2\omega^4}{12\pi\varepsilon_0 c^3} = \frac{q^2\omega^2}{6\pi\varepsilon_0 mc^3}E$$

Satisfies the correspondence principle as expected.

* Selection Rules: when does $\vec{\wp} = \langle\psi_b|q\vec{r}|\psi_a\rangle = 0$?

for $\mathcal{H}$ symmetric, $\psi = f(r)Y_{\ell m}(\theta,\phi) \rightarrow \vec{\wp} = q\langle n'\ell'm'|\vec{r}|n\ell m\rangle$

+ rules for m, m': $[L_z, x] = i\hbar y$, $[L_z, y] = -i\hbar x$, $[L_z, z] = 0$

a) $0 = \langle n'\ell'm'|[L_z, z]|n\ell m\rangle = \langle n'\ell'm'|L_z z - zL_z|n\ell m\rangle$

$= \hbar\langle n'\ell'm'|m'z - zm|n\ell m\rangle = \hbar(m-m')\langle n'\ell'm'|z|n\ell m\rangle$

thus $\wp_z = 0$ unless $m \neq m'$

b) $[L_z, x_\pm] = \pm\hbar x_\pm$ $x_\pm = x \pm iy$

$$\langle n'l'm'|[L_z, x_\pm]|nlm\rangle = \langle n'l'm'|L_z x_\pm - x_\pm L_z|nlm\rangle$$

$$\langle n'l'm'|\pm\hbar x_\pm|nlm\rangle = \pm\hbar(m'-m)\langle n'l'm'|x_\pm|nlm\rangle$$

thus $p_x \pm i p_y = 0$ unless $m' = m \pm 1$

selection rules for m: $\boxed{\Delta m = \pm 1, 0}$

+ rules for l: $[L^2,[L^2,\vec{r}]] = 2\hbar^2(\vec{r}L^2 + L^2\vec{r})$

$$\langle n'l'm'|[L^2,[L^2,\vec{r}]]|nlm\rangle = \hbar^2\left[l'(l'+1) - l(l+1)\right]\langle n'l'm'|[L^2,\vec{r}]|nlm\rangle$$

$$\| \qquad = \hbar^4\left[l'(l'+1) - l(l+1)\right]^2\langle n'l'm'|\vec{r}|nlm\rangle$$

$$2\hbar^2\langle n'l'm'|\vec{r}L^2 + L^2\vec{r}|nlm\rangle = 2\hbar^4\left[l'(l'+1) + l(l+1)\right]\langle n'l'm'|\vec{r}|nlm\rangle$$

$$\hbar^4\left\{\left[\underbrace{l'(l'+1) - l(l+1)}_{(l'+l+1)(l'-l)}\right]^2 - \underbrace{2\left[l'(l'+1) + l(l+1)\right]}_{(l'+l+1)^2 + (l'-l)^2 - 1}\right\}\langle n'l'm'|\vec{r}|nlm\rangle = 0$$

$a^2b^2 - (a^2+b^2-1) = 0$

$[a^2-1][b^2-1] = 0$

selection rule for l: $\underbrace{[(l'+l+1)^2-1]}_{\text{can't be } 0}[(l'-l)-1] = 0 \Rightarrow \boxed{\Delta l = \pm 1}$

+ rules for m_s: $\boxed{\Delta m_s = 0}$