L85-Review

Friday, April 29, 2016 08:50

* Final Exam – cumulative from whole semester
  - must solve 4/5 problems (see midterm)
    each with a calculation & conceptual questions

4) Spin & Angular momentum
   Quantum numbers & ladder operators.
   Pauli Matrices, diagonalization,
   combination of 2 spin-½ systems;
   evolution of spin system in B-field.

5) Identical particles
   Symmetrization of multiparticle wave fn: ψ_{even}, ψ_{odd}.
   Atomic configuration
   Statistical counting of microstates from single-particle states.
   Description of M-B, F-D, BE distributions., Bose condensation.
   Blackbody spectrum

6) Time-independent perturbation theory
   solve nondeg. & degenerate first-order energy & eigenstates
   using either operator wave functions or perturbation matrix
   [hyper] fine structure: show magnetic interaction produces $\vec{L}\cdot\vec{S}$ coupling
   identify states with good $\vec{L}\cdot\vec{S}$, $\vec{I}\cdot\vec{J}$
   Zeeman effect: states & energies in low/strong field limit.

7) Variational Principle
   problem to minime energy of parametrized state.

9) Time dependent perturbation theory:
   calculate evolution $[c_a(t), c_b(t)]$ to first order
   calculate a transition probability:
   derive relation between A, B coefficients.
   Explain Breit-Wigner line shape, selection rules.

11) Scattering

explain the relation between various waves, amplitudes, and cross sections

solve for partial wave scattering amplitude.

* Fermi's golden rule: connection between Born Approximation & time dependent perturbation theory

$$R_{i\to f} = \frac{2\pi}{\hbar}\left|\langle f|\mathcal{H}'|i\rangle\right| \rho(E') = \frac{\sigma \cdot v}{V} \qquad \rho(E') = \frac{dn}{dE'} = \frac{V}{(2\pi\hbar)^3}\frac{4\pi p^2 dp\, d\Omega}{dE'}$$

$$\sigma = \frac{2\pi}{\hbar v}\left|\langle f|\mathcal{H}'|i\rangle\right|^2 \rho(E') V = \frac{V^2 E'^2}{(2\pi)^2(\hbar c)^4}\left|\langle f|\mathcal{H}|i\rangle\right|^2 \Delta\Omega$$

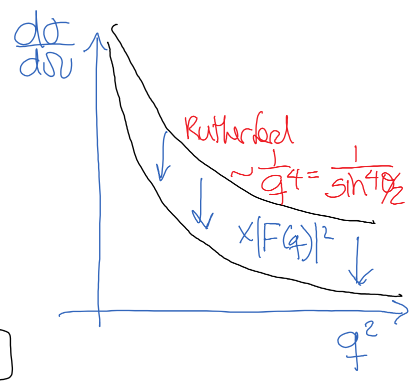
– compare to rate of stimulated emission

$$R_{i\to f} = \frac{\pi}{3\varepsilon_0\hbar^2}\langle f|e\vec{r}|i\rangle \rho(\omega_0) = \frac{\pi|\gamma_0|^2\rho(\omega_0)}{3\varepsilon_0\hbar^2}$$

– scattering is a transition from one state to another

$$f(\theta,\phi) = \frac{-m}{2\pi\hbar^2}\langle f|V|i\rangle = -\frac{m}{2\pi\hbar^2}\int d^3\vec{r}\, e^{i\vec{q}\cdot\vec{r}} V(\vec{r})$$

* Elastic scattering & form factors:

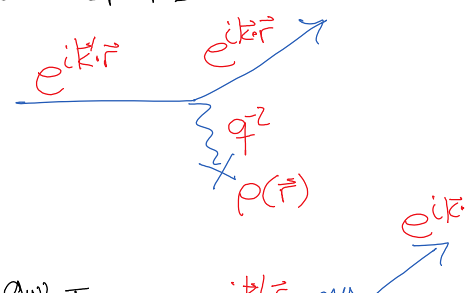


$$\langle f|\mathcal{H}'|i\rangle = \frac{1}{V}\left|\langle e^{i\vec{k}'\cdot\vec{r}}|e\phi(\vec{r})|e^{-i\vec{k}\cdot\vec{r}}\rangle\right|^2 = \int e\phi(\vec{r})e^{i\vec{q}\cdot\vec{r}}d^3\vec{r}$$

$$= \int e\left[\nabla^{-2}\rho(\vec{r})/\varepsilon\right]e^{i\vec{q}\cdot\vec{r}}d^3\vec{r} = \frac{e}{q^2\varepsilon}F(\vec{q})$$

$$q^2 = (p'-p)^2 = p'^2 + p^2 - 2pp'\cos\theta = 4p^2\sin^2\theta/2 \qquad [p=p']$$

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{Z^2\alpha^2(\hbar c)^2}{4E^2\sin^4\theta/2}}_{\text{Rutherford cross. sec.}} \cdot \underbrace{|F(q)|^2}_{\text{form factor}}$$

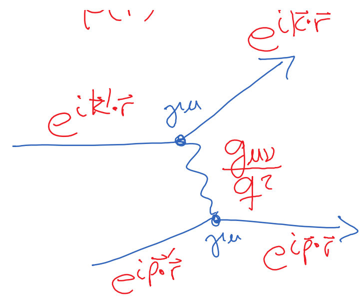


QED: $\langle f|\mathcal{H}'|i\rangle \Rightarrow$ invariant amplitude $\mathcal{M} = J_\mu \frac{g_{\mu\nu}}{q^2} J_\nu$

massless photon γ has propagator $\frac{g_{\mu\nu}}{q^2}$

connecting 2 scattering currents:

$J^\mu = \langle f | e\gamma^\mu | i \rangle \qquad \gamma^\mu \sim$ spin.



Impulse expansion: $G(q) = \dfrac{1}{q^2+k^2} \sim \dfrac{e^{ikr}}{r}$ propagator for T.I.S.E.

Massive particles: $G(q) = \dfrac{1}{q^2-m^2} \sim \dfrac{e^{-mr}}{r}$ Yukawa potential