Simulating Particle Beamlines using Python

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Abstract

For particle accelerators to properly store and record data from particles, the incoming beam of particles must be centered and, at times, be manipulated to redirect its direction of travel. This action of influencing particle beams can be simulated using computer programming. I have used the programming language Python to simulate these particle beams and the effects of various "magnetic lenses" on the beam's path along with filtering out unwanted particles from the beam. This program will aid additional research by increasing the confidence in experiments using particle beams before their testing has begun.

Establishing the Simulation

The simulation relies on solving the equations of motion for particles in the presence of beamline elements. The two basic elements are the Drift and Lens

- Drift - empty space, typically a vacuum
  - A particle will not experience any external forces
- Lens - a magnetic quadrupole
  - A thin quadrupole can be approximated as a lens with a focal length

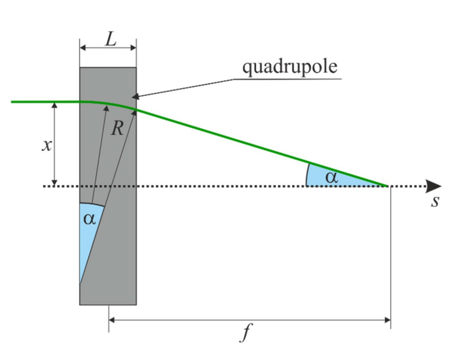

Fig. 1: Optical interpretation of beamline quadrupole[1]

$$f = \frac{1}{k \cdot l_q}$$

$$k = \frac{g}{B \cdot \rho}$$

Eq. 1-2: Thin quadrupole approximation[2]

The particle's position along with beamline elements can be represented as matrices. This allows for Python to quickly calculate estimated motion. These equations were later expanded to 3D.

Lens

$$\begin{bmatrix}\hat{x}\\ \hat{x}'\end{bmatrix} = \begin{bmatrix}1 & 0\\ -\frac{1}{f} & 1\end{bmatrix}\begin{bmatrix}x\\ x'\end{bmatrix}$$

Drift

$$\begin{bmatrix}\hat{x}\\ \hat{x}'\end{bmatrix} = \begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\ x'\end{bmatrix}$$

Eq. 3-4: Matrix representation of equations of motion

Beamline Arrangements

FODO Cells

A common arrangement found in particle beamlines is a FODO cell. The cell consists of a focusing quadrupole (F), drift space (O), defocusing quadrupole (D), and a final drift space (O).

$$\begin{bmatrix}\hat{x}\\ \hat{x}'\end{bmatrix} = \begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{f} & 1\end{bmatrix}\begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -\frac{1}{f} & 1\end{bmatrix}\begin{bmatrix}x\\ x'\end{bmatrix}$$

Eq. 5: Matrix representation of a FODO cell

Triplet Cells

Another arrangement of components is the triplet cell. This arrangement has three separate quadrupoles separated by equal drift spaces. The triplet cells we studied had a middle quadrupole with a focal length that was a negative half of the focal length of the outer two focusing quadrupoles.

$$\begin{bmatrix}\hat{x}\\ \hat{x}'\end{bmatrix} = \begin{bmatrix}1 & 0\\ \frac{1}{-f} & 1\end{bmatrix}\begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{2f} & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{-f} & 1\end{bmatrix}\begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\ x'\end{bmatrix}$$

Eq. 6: Matrix representation of a triplet cell

Particle Tracing

The program presents the opportunity to represent particle simulations in a variety of ways. The two most common visual simulation results used in this research project are the scatter and distribution plot.

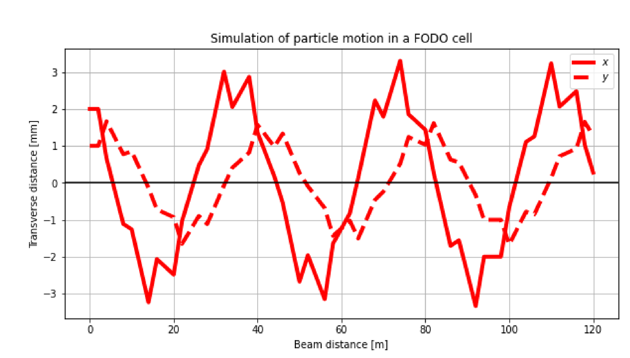

Fig. 2 (left): 2D representation of particle motion

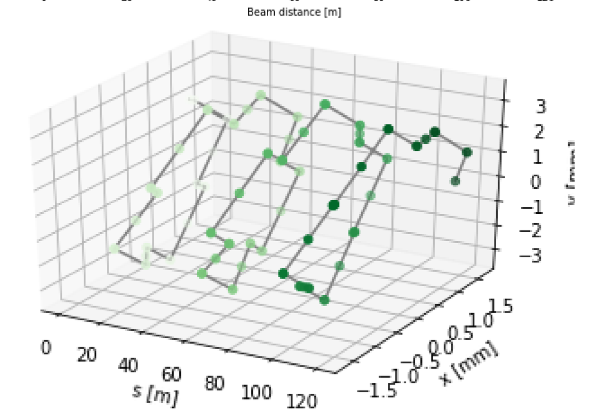

Fig. 3 (left): 3D representation of particle motion

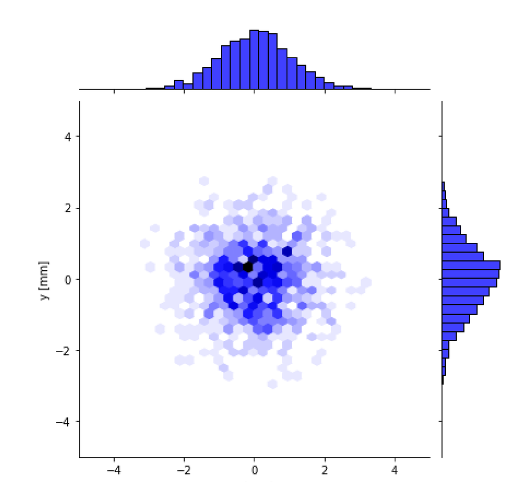

Fig. 4 (above): Transverse distribution of 1000 particles before entering beamline

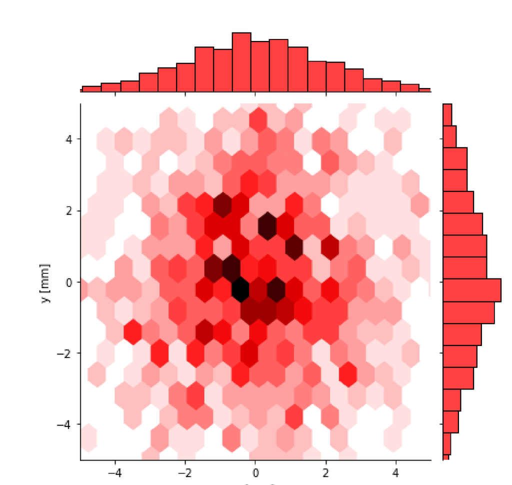

Fig. 5 (above): Transverse distribution of 1000 particles after entering beamline

Dipoles

Another beamline element used to direct particles is the magnetic dipole

- Used to bend the particle beam longitudinally
- Useful in beamlines that curve, such as storage rings

$$\begin{bmatrix}\hat{x}\\ \hat{x}'\end{bmatrix} = \begin{bmatrix}cos(\frac{l}{\rho}) & \rho sin(\frac{l}{\rho})\\ -\frac{1}{\rho}sin(\frac{l}{\rho}) & cos(\frac{l}{\rho})\end{bmatrix}\begin{bmatrix}x\\ x'\end{bmatrix}$$

Eq. 7: Matrix representation of a dipole

Expanding Matrices

To no surprise, real particle beamlines exist in three dimensions. Therefore, the beamline element matrices must be expanded to account for the additional dimension.

Lens

$$\begin{bmatrix}1 & 0 & 0 & 0\\ \frac{-1}{f} & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & \frac{1}{f} & 1\end{bmatrix}$$

Drift

$$\begin{bmatrix}1 & L & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & L\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Dipole

$$\begin{bmatrix}cos(\frac{l}{\rho}) & \rho sin(\frac{l}{\rho}) & 0 & 0\\ -\frac{1}{\rho}sin(\frac{l}{\rho}) & cos(\frac{l}{\rho}) & 0 & 0\\ 0 & 0 & 1 & l\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Eq. 8,9,10: Matrix representation of lens, drift, and dipole transfer matrices

Triplet Stability Condition

The symplectic nature that the transfer matrices possess allows for researchers to calculate whether a particle beam will be stable. The stability condition arises from the trace of the transfer matrix.

$$Tr(M) \leq 2$$

$$\downarrow$$

$$0 \leq \frac{L}{f} \leq 1$$

Eq. 11[2],12: Derivation of stability condition

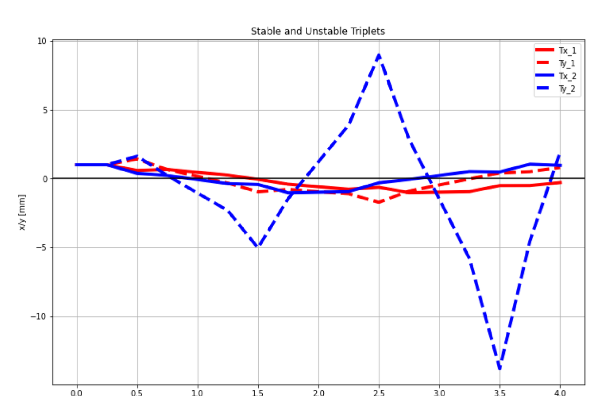

Fig. 6: Motion of identical particles in stable and unstable triplet cells

Single Meter Restriction

To compare the effects the two cells have on identical particles, a FODO and triplet cell were simulated given the following parameters:

- supply equivalent effective focal lengths
- occupy the same amount of space

It was found that the triplet cell was more effective at focusing the particle in both the x and y directions over a short distance.

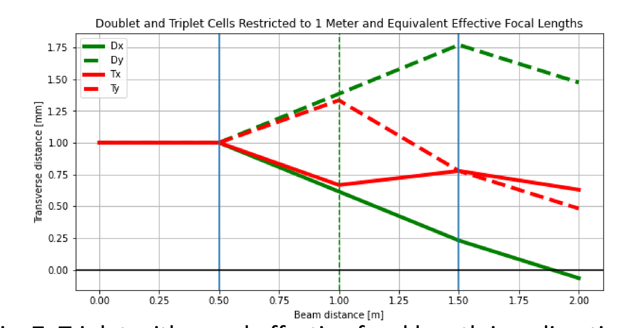

Fig. 7: Triplet with equal effective focal length in y direction

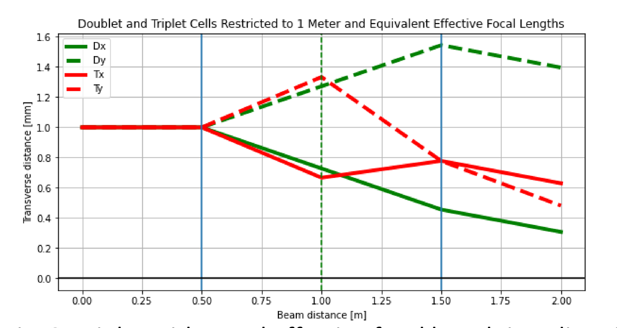

Fig. 8: Triplet with equal effective focal length in x direction

Infinite Effective Focal Length

Effective focal lengths depend on the individual focal lengths of each lens and their separation distances

- Specific values can be chosen to create a system of FODO cells with an infinite effective focal length
- No net movement in transverse plane
- Two FODO cells for 180° rotation or four FODO cells for a complete 360° rotation

$$f = \frac{L}{\sqrt{2}}$$

Eq. 13: Parameter that results in 360°rotation

x4 FODO's

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

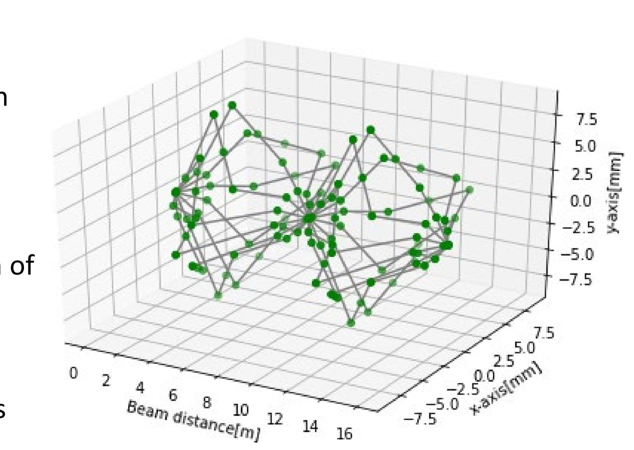

Fig. 9: 3D simulation of particles traveling through 4 FODO cells

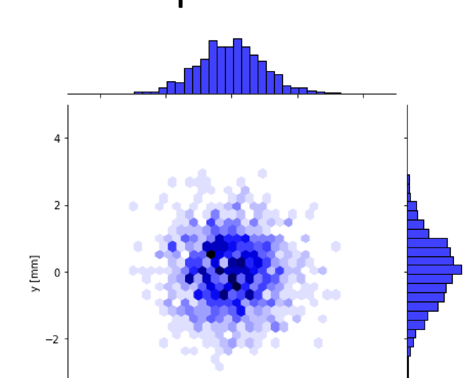

Fig. 10: Distribution before

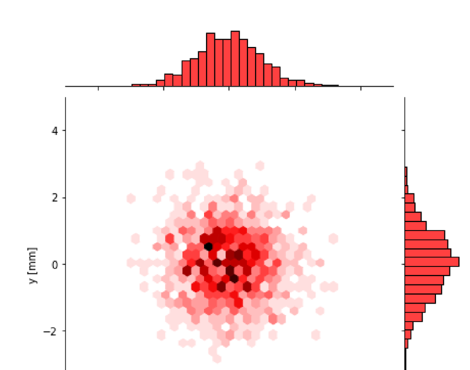

Fig. 11: Distribution after

Future Plans

The next steps towards developing a more cohesive particle beamline simulation is to implement individual particle momentum. This allows for the simulation to account for effects that arise from momentum dispersion such as particle filtering.

References and Acknowledgements

[1] Wolfgang Hillert. Transverse linear beam dynamics, 2021

[2] Hao Research Group, Transverse dynamics, 2021

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