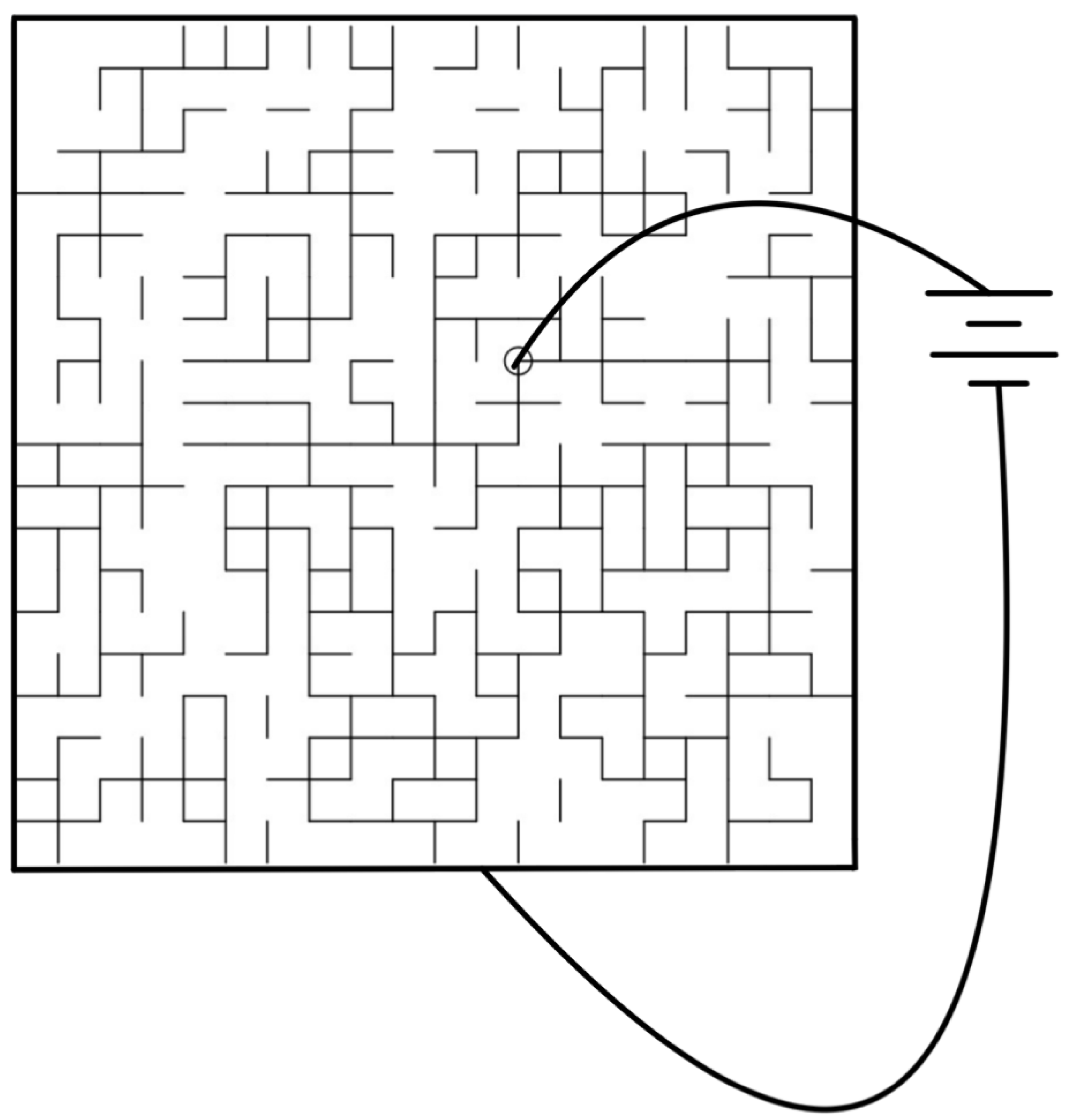
Percolative Conduction and The Distance Function

Matthew Golden[1,2], Dr. Joe Straley[1]

[1]Department of Physics, University of Kentucky
[2]Department of Physics, Bellarmine University

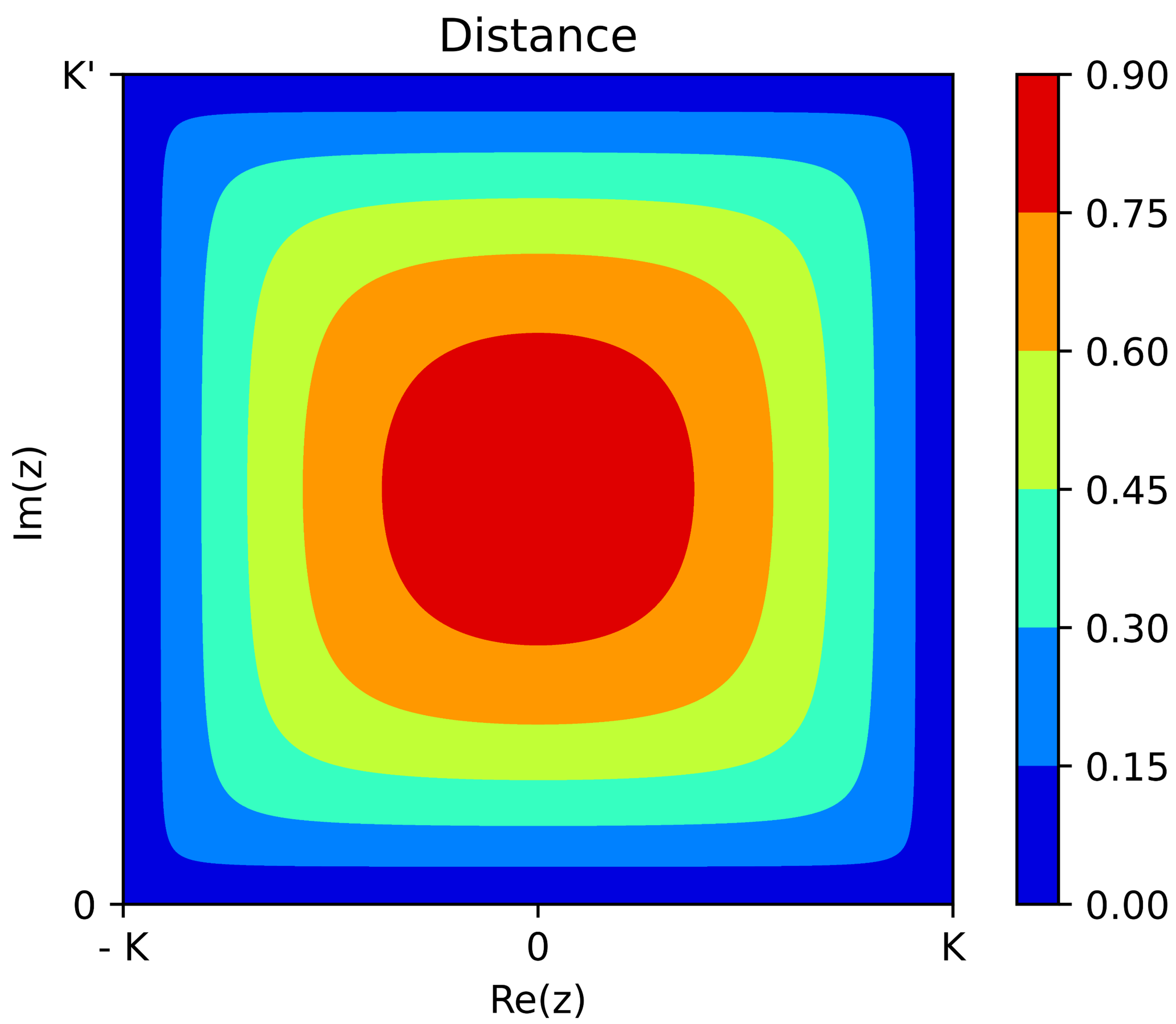
Introduction

We are modeling a conductive system as a percolation network to study critical phenomena. In our model of percolation, a network in a square lattice consists of randomly chosen "links" which are joined “nodes”.
In our conductive system the links represent conductors, with specified conductance; equal to 1(link) or 0(no link). The boundaries of the square are grounded, allowing current to exit. At a chosen node inside the boundaries the voltage is specified to be V = 1. The voltages on the other sites generate currents on the conductive links.
We are interested in how does the conductance depend on the distance from the edge?



Methods

The program we used to study this model, written by Dr. Straley, has a variable p that is the fraction of internal bonds that unit conductance. The percolation threshold for the square lattice is exactly $p_c = \frac{1}{2}$, so we will run our simulations with $p = \frac{1}{2}$. We ran the program at 500 random coordinates, with square lattice size N = 101.



$$Distance = \left[\frac{Im\left[sn\left(\left[\frac{2K}{N}\right]z\middle|k\right)\right]}{\left|cn\left(\left[\frac{2K}{N}\right]z\middle|k\right)\times dn\left(\left[\frac{2K}{N}\right]z\middle|k\right)\right|}\right]$$

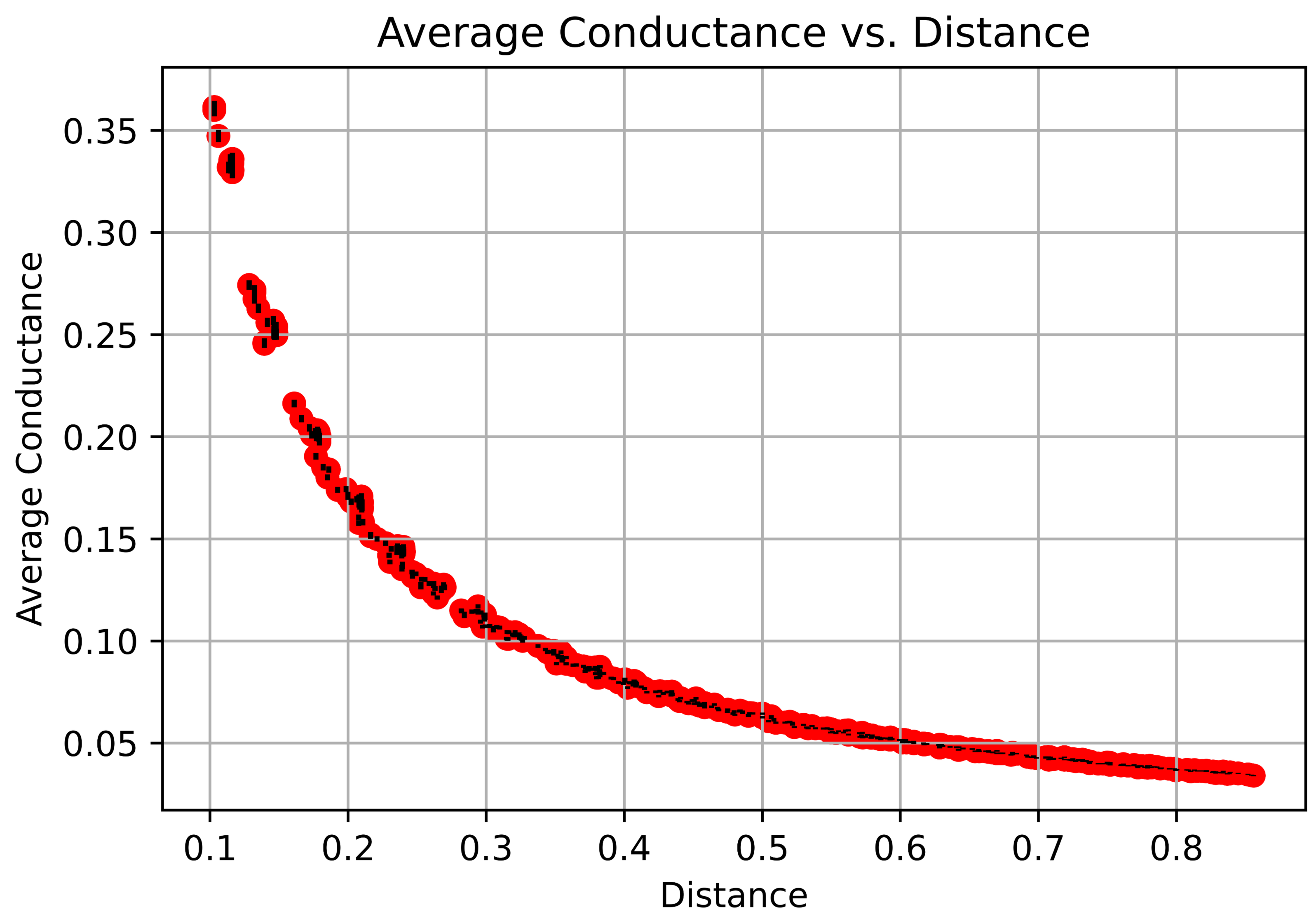
Distance Function

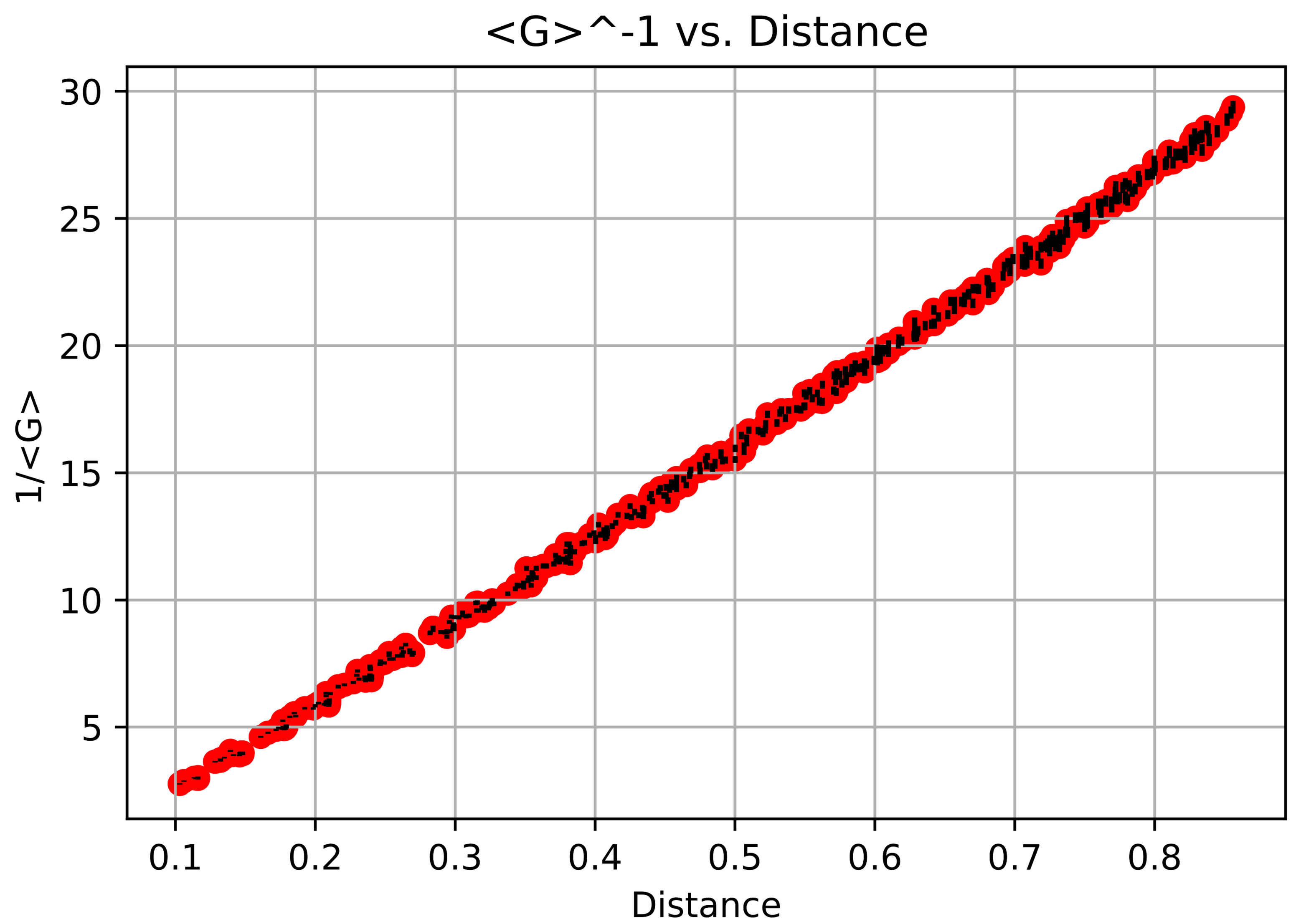
To find the relationship between the average conductance and the distance to the boundary, we want the "distance" that explains between a point in the square and the boundary, that treats all sides the same and makes sense from the point of view of conformal transformations. Here $z = (x + i(y + K))$ is a complex variable defined on the rectangle, and sn, cn, and dn are the Jacobian elliptic functions.

Conclusions

The case of the square lattice with all sides conducting, we have seen that the relationship between the average conductance and the size of the lattice by a power law hold true, and we have seen the same for the conductance and the distance to the boundary.

$$G = \left[\frac{Im\left[sn\left(\left[\frac{2K}{N}\right]z\middle|k\right)\right]}{\left|cn\left(\left[\frac{2K}{N}\right]z\middle|k\right)\times dn\left(\left[\frac{2K}{N}\right]z\middle|k\right)\right|}\right]^{\frac{-t}{\nu}}$$





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