Analysis of RR Lyrae Variable Stars

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Abstract

RR Lyrae stars have been known for being radial pulsators for quite some time. RR Lyrae are in the instability strip of the Hertzsprung-Russell Diagram (HRD) and can be classified as RRab, RRc, and RRd. Most of the analysis is done with RRc stars which are characterized as first-overtone pulsators. Interestingly, many RRc stars also exhibit a non-radial pulsation mode (denoted Fx) and it is currently not known what the mechanism is that drives this non-radial mode. Investigations into this mechanism has led to a possibility that the non-radial pulsation mode is driven by magnetic fields built up by turbulent flow in the hydrogen and helium ionization zones[1].Moskalik in 2015 using Kepler data, suggested that the Fx mode and F1 mode were anti-correlated[2]. later Dziembowski in 2016 claimed that the (1/2)Fx sub-harmonic may actually be the non-radial mode instead of a sub-harmonic[3]. We test these claims using light curve data on XX Dor, an RRc variable star with a large amount of data collected on it.

1 Introduction

It is known that Radial pulsation works through the κ-mechanism (where κ denotes the opacity).In this process, the partially ionized helium zone collapses under gravity below equilibrium which leads to an increase of opacity but the temperature remains relatively constant because opacity is not obeying Kramer's law:

$$\kappa \propto \frac{\rho}{T^{3.5}} \tag{1}$$

Instead the opacity is dominated by electron scattering and thus the opacity is independent of temperature. The increased opacity then traps heat from the radiative zone building pressure that eventually pushes the partially ionized helium zone outwards past equilibrium. Then the Helium layer cools allowing recombination of electrons to the Helium ions and lowers the opacity. Again the Helium zone begins to collapse under gravity falling past the equilibrium point and the cycle repeats itself.[4]

Over time evidence has been gathered that suggests that some but not all RRc stars have an additional pulsation mode unaffiliated with the radial pulsation with frequency Fx $\sim$ F1 / 0.61. Additionally, Dziembowski claimed through theoretical models that this mode must be a non-radial mode with Legendre polynomial degree of $l = 8$[3]. Further more, Moskalik attempts to characterize this Fx mode using four Kepler stars and found when compared to the Primary modes amplitude that Ax and A1 (the amplitudes of Fx and F1 respectively) were anti-correlated with each other[2]. For the Fx mode, finding how it changes and whether there is any periodicity in that change can help characterize the Fx mode which can help narrow down what the driving mechanism is and validate some of the previous claims. I will be using Period04[5], a computer program used for statistical analysis of large astronomical time series using fourier transforms to get frequencies, amplitudes, and phases from light curves. Using various light curve observations taken by the Transiting Exoplanet Survey Satellite (TESS)[6] I can upload time strings of RRc light curves into Period04.

Using these light curves I split the data of a sector (one time string file) into four sections with 20 cycles in each group if possible. I go through a process of prewhitening which filters out the primary radial pulsation mode and begins constructing a model with that.I then pick out the Fx mode and record its amplitude and with enough data we can see how the Fx mode is changing. I also closely analyze the sub-harmonics of Fx which is (1/2)Fx and (3/2)Fx because of a possibility of a relationship between them. Other combinations of Fx also exist in the fourier spectrum generated by Period04 but they will not be considered.

2 Procedure

As previously described I am using Period04 to conduct analysis on RRc variable stars. A particular star I am interested in is XX Dor were we have 7 months of observations plus an additional 12 months of observations done on it. XX Dor features an Fx mode which commonly splits into multiplets as shown below:

Before even using Period04 the light curves are in fluxes versus Julian date but it is convention to use magnitudes to represent light intensities. There is also a known issue with Period04 if the Julian dates are not initialized to zero then the phases from Period04 will not come out correctly. This might be because Period04 generates a model of the light curve starting from zero and so it calculates phases starting from zero days. Typically, Julian dates start at a 1000 days or more roughly speaking. I do all of this using a python program that accepts a data file, sets the Julian date approximately to zero by cutting off the number left of the decimal point, and doing the appropriate conversions of flux and flux errors to magnitudes and magnitude errors. Additionally, the program also sets every sector, which is one data file containing about a month of observations, to the same zero-point. This is to ensure their is no deviation of amplitude values because of sectors not having the same relative zero-point

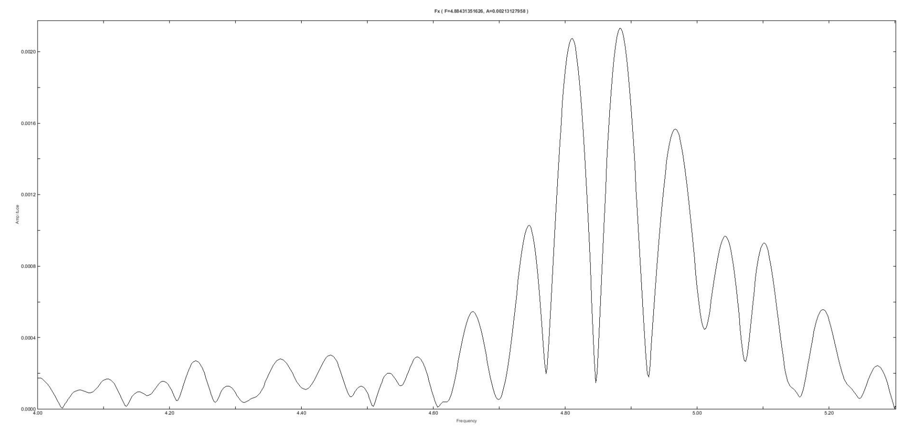

Figure 1: Fourier spectrum after prewhitening F1 (radial mode) out showing the multiple peaks corresponding to Fx

like in this figure:

I then split these sectors into smaller groups but caution is necessary when doing so. If the data is split up too much and analyzed individually then the peaks generated by the Fourier transform widen, taking the form of a Gaussian distribution. If these peaks widen enough they can overlap with each other especially in the circumstance of multiplets with the peaks very close together. A minimum of about 18 cycles of the light curve is necessary in order to avoid this problem. Here is an example of what a split sector might look like:

On the other hand, we want to be able to split the data and measure the amplitude of Fx in each group to see how it changes with time. I then analyze each group individually beginning with the prewhitening process. First the Fourier transform gives us F1 which is then fitted by Period04 using this formula:

$$Z + \Sigma A_i sin(2\pi(\Omega_i t + \Phi_i)) \qquad (2)$$

where Φ_i denotes the ith phase, Ω_i denotes the ith frequency, and Z denotes the zero point. We need F1 and its integer multiple harmonics in order to complete prewhitening. Since there are an infinite number of harmonics we utilize a signal-to-noise ratio (SNR) built into Period04 that tells me when the signal is no longer distinguishable from noise and thus could be ignored. Once an SNR less than 5 is achieved then enough harmonics have been calculated and the prewhitening process is done.

Once prewhitening is complete I move on to Fx. By using the Fourier transform to calculate residuals we find Fx which is typically the largest peak still left over, however, as we will see sometimes it is not the largest. This is due to Fx sometimes splitting into multiplets as shown in Fig.1. If the Fx mode is split into multiplets I collect all of them in Period04 and I add the amplitudes together treating Fx as a superposition of this smaller amplitudes. I also

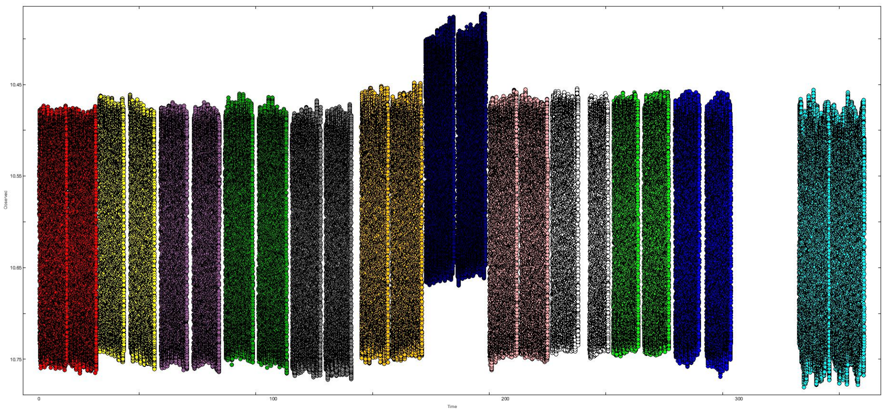

Figure 2: A real example of sectors highlighted in different colors being significantly shifted from each other(particularly the 7th sector in this case)

average the frequencies to compare the values with singular frequencies in the non-multiplet case. Using the same methods, I then find the 1/2Fx and 3/2Fx modes and I use the SNR to test if their amplitudes are distinguishable from noise using the same ruling as before. Once I have obtained all of the frequencies I use Period04 and run a Monte Carlo simulation to compute errors in the amplitudes and phases. I keep the frequencies fixed since they are not what we a primarily interested in. After I have completed this process I export the frequencies, amplitudes, and phases together, the non-initialized Julian dates, and the errors in three separate files. I repeat this entire process for every grouping until everything in the data set has been analyzed and then feed the data files into a new python script. This python script generates a table of different values and does all non-Period04 calculations such as averaging the frequencies and adding the amplitudes for multiplets of Fx. It additionally averages the Julian dates for each group for the purposes of plotting the amplitudes. We're also interested in the ratios Ax/A1, (1/2Ax)/A1, and (1/2Ax)/Ax (the A's denote the amplitude) which python can also compute.

3 The Fourier Transform

As stated above Period04 utilizes a Fourier transform to find the different modes associated with a light curve. In particular we use a discrete Fourier transform of the form:

$$f(\omega) = \sum_{j=1}^{N} X(t_j) e^{-i\omega t_j} \tag{3}$$

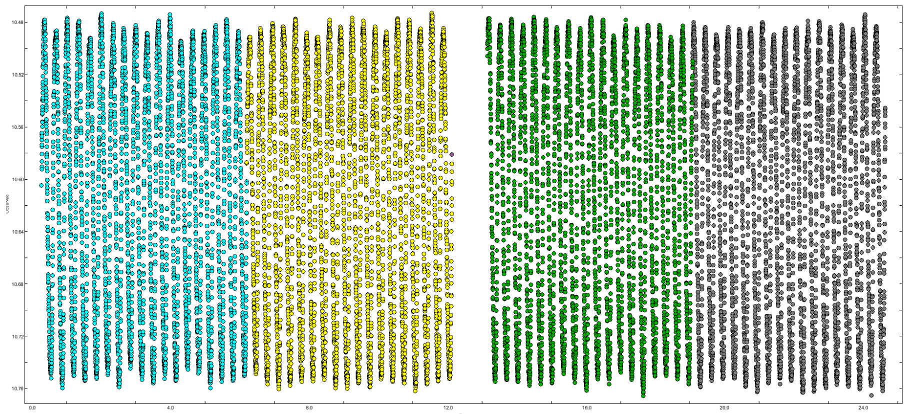

Figure 3: A sector after appropriate divisions have been made

Where X denotes the data at time t_j and ω denotes the frequency. In this discrete Fourier transform the components leave only specific frequencies while the other's cancel out in the summation[4]. As the number of observations approaches infinity i.e. $N \to \infty$ the remaining frequencies become delta functions. Together these delta functions $\delta_j(\omega)$ form a function known as a Dirac comb shown below:

Of course we do not have infinite observations so N is finite and the Fourier spectrum peaks take on the appearance of Gaussian distributions. The less data that we have, the wider these Gaussian distributions get which can create overlap with other peaks and complicate analysis as stated earlier.

4 5 Month Observation Results

We used data gathered by TESS that corresponds to about 7 months worth of observations. These observations however, were not "continuous" and by that we mean there is a significant gap between the first 5 months of data and the last 2 as shown below:

The gap size between sectors and halfway in between sectors is negligible. Following the procedure we divide each sector into smaller groups such that each group contains at minimum 18 cycles of the primary light curve. Typically this means we can divide each sector into 4 groups, however, this was not always possible. Sector 4 in particular was divided into three groups before grouping was done as shown in figure 6 below:

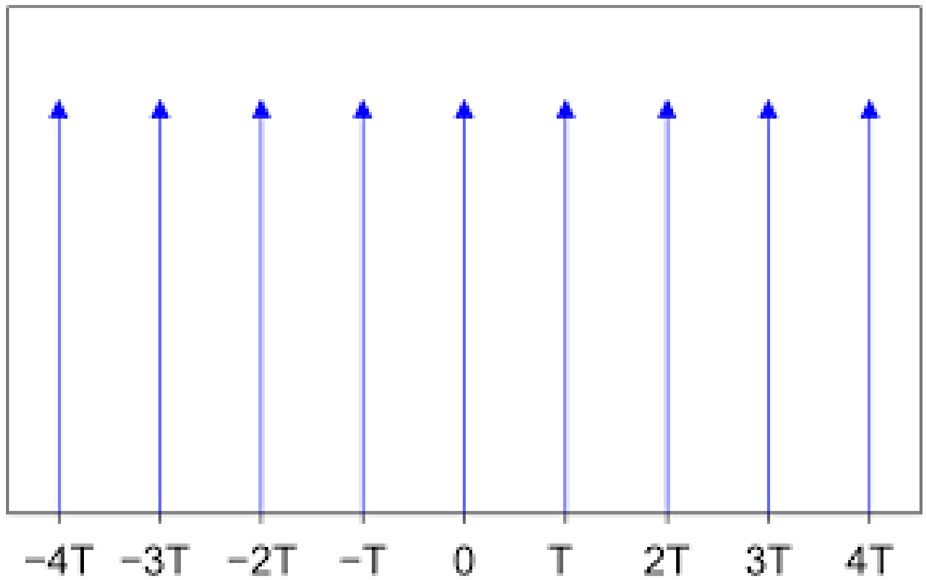

Figure 4: An example of what a Dirac Comb looks like visually. This image was acquired from Wikipedia

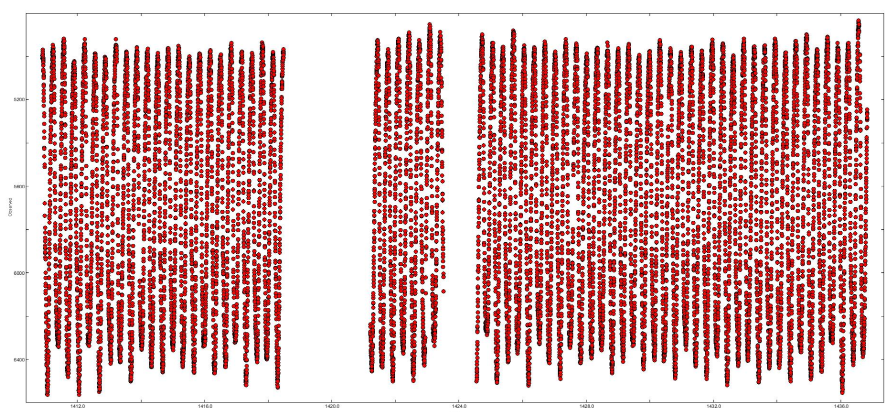

Figure 6: Sector 4 of the 7 month observations by TESS

As a result of this split in the data the middle section was omitted from the analysis and the two remaining were analyzed as is since any smaller groupings would be below the 18 cycle minimum we're looking for. Data on the Fx mode, F1, (1/2)Fx, (3/2)Fx, and Fx+F1 modes had been collected but due to insufficient data for (3/2)Fx, and Fx+F1 no plots of these were created. Below is Fx and F1 plotted versus time:

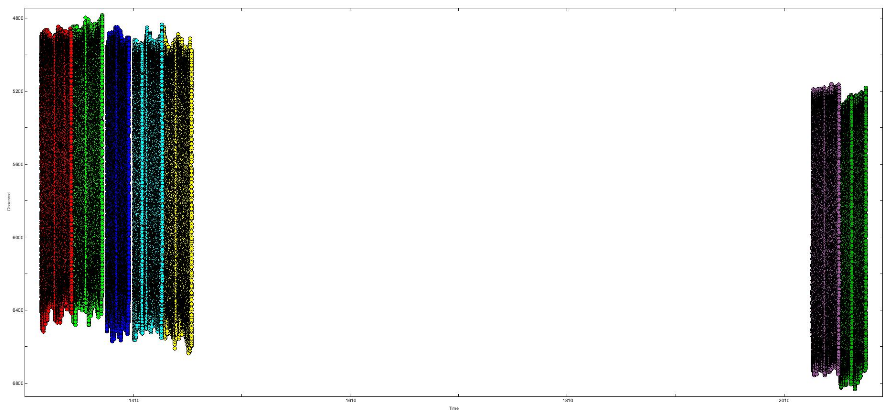

Figure 5: Each color refers to a sector (a months worth of data). The last two months were omitted because of the significant gap. Light curves are in fluxes.

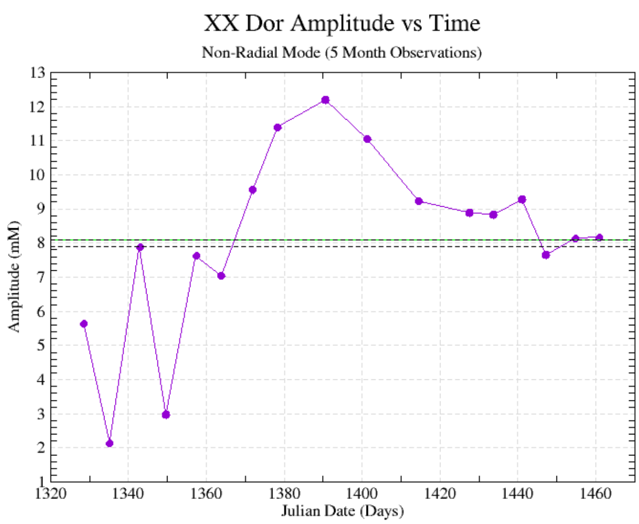

Figure 7: XX Dor Fx mode amplitude over time. (black dashes indicate the average error)

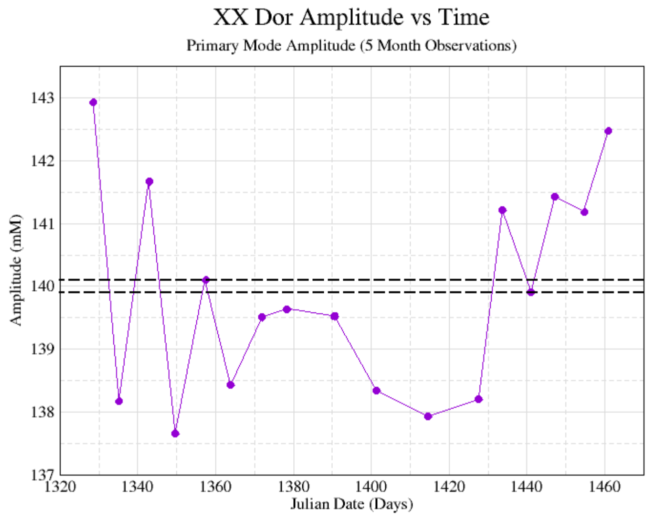

Figure 8: XX Dor F1 mode amplitude over time. (black dashes indicate the average error)

Here we see the Fx mode changes rapidly between groupings at the start suggesting a wave like pattern. Fx varies from 2mM to a maximum of 12mM overall. When Comparing Ax (the amplitude of Fx) and A1 they seem like they could be correlated at 1330-1360 days and anti-correlated 1360-1425 days were Ax appears to be large in comparison to A1. However, more rigorous testing will need to be done to determine if there is a real correlation between A1 and Ax. The next comparison was done between Fx and the (1/2)Fx sub-harmonic as shown in figure 9 below. The reason for this is similar to Moskalik's findings [2]. It would be an interesting characteristic if the Fx mode shared energy with its sub-harmonic or if the sub-harmonic was on average larger than Fx.Dziembowski[3] in 2016 made the claim that the (1/2)Fx sub-harmonic may not be a sub-harmonic at all and instead may be the true non-radial mode.

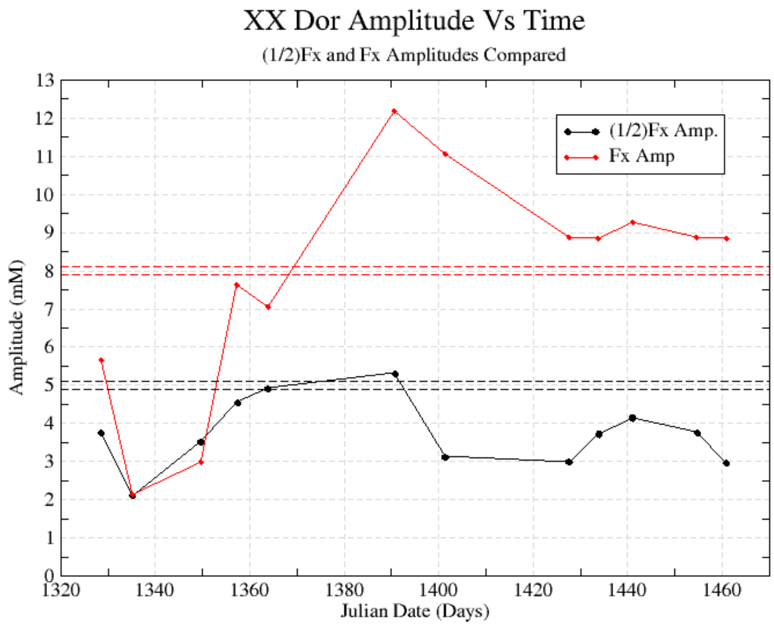

Figure 9: XX Dor 5 month Observation Sub-harmonic and Fx modes(Dashed lines indicate respective average error)

Here at the beginning it would seem the Fx and (1/2)Fx modes are correlated until they begin to diverge significantly at 1350 days. At that point we see Fx's amplitude increase dramatically but a correlation can still be seen between the two. Again more rigorous correlation tests will need to be done in order to quantify how correlated Fx and (1/2)Fx are.

5 10 Month Observation Results

TESS also has observations over the course of a year done after the previous 7 month observations. This is far more data than what you would typically expect for one star which makes XX Dor particularly useful for analyzing long-term changes. Unfortunately, sector 7 was heavily zero-point shifted as seen in figure 2. Once analysis was initially done on this sector it was found that A1 varied significantly from all other sectors in the data set which could mean something is wrong with the data and is not a real change occurring. More work will need to be done to determine this and is thus left out of our analysis for now. The 12 sector was similar to sectors 6 and 7 in the other data set and was separated from the rest of the data with a large gap as shown in figure 2. Because of these reasons we omitted both of these sectors without a significant loss in data leaving us with 10. Here are the results of the Fx and F1 amplitudes plotted similarly to figures 7 and 8:

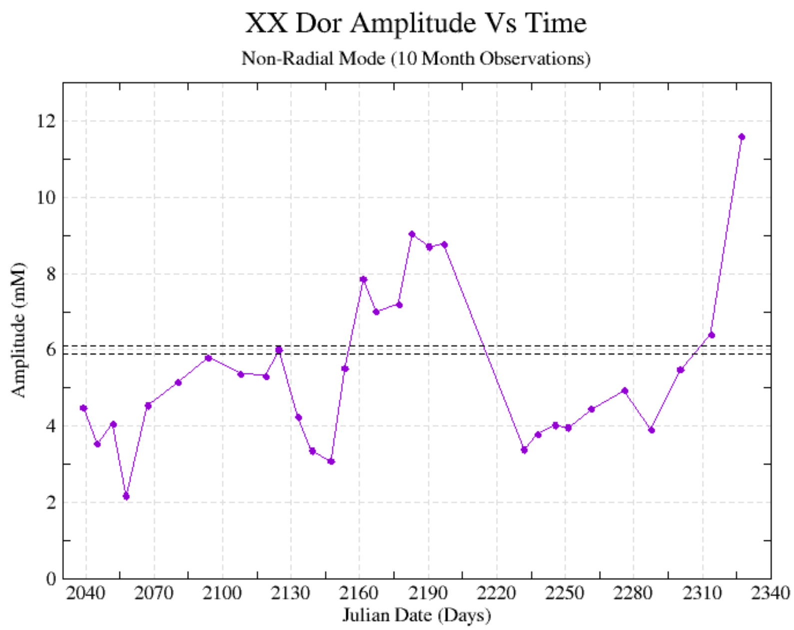

Figure 10: XX Dor 12 month Observation Fx mode amplitude over time(Dashed lines indicate respective average error)

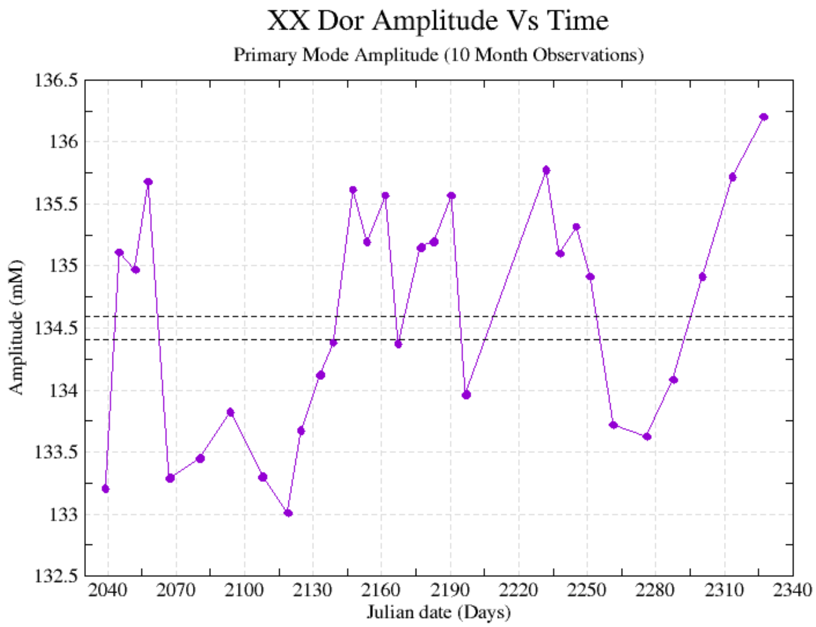

Figure 11: XX Dor 12 month Observation F1 mode amplitude over time(Dashed lines indicate respective average error)

Here we can see with more data available that Ax appears to be changing with a somewhat wavelike pattern. When compared with A1 we see both maximize towards the end of observations 2340 days. At the start it looks like their could be anti-correlation between Ax and A1 from 2030 days to 2070 days so roughly a 40 day period. However, no further anti-correlation can be seen, in fact, it would seem they are correlated for the remainder of the data. More work will need to be done to really quantify how correlated A1 and Ax are. Another interesting note is with the variation of A1. A1 clearly varies less than A1 in the 7 month data and appears to be shifted down in magnitude from the 7 month data. This is very interesting because noticeable changes over long periods of time are not normal for A1 in most RRc stars.

Below in figure 12 we see the sub-harmonic (1/2)Fx compared with Fx in amplitude over time whenever (1/2)Fx was detectable:

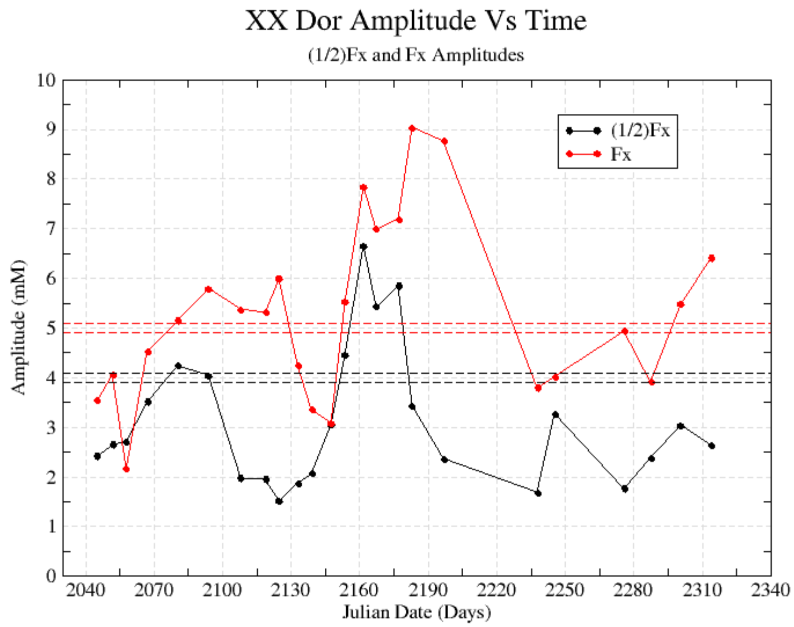

Figure 12: XX Dor 12 month Observation (1/2)Fx and Fx modes amplitude over time(Dashed lines indicate respective average error)

Interestingly we see (1/2)Fx mimic the behavior of Fx implying they are correlated with Fx being scaled up by a factor. Further investigation will be needed to get a better understanding of the relationship between (1/2)Fx and Fx and will be explored in the future.

6 Conclusion and Future Plans

XX Dor experiences strange shifts in its radial modes amplitude between data sets which could be implying a longer-term physical change in the star. New questions would then arise such as is this change periodic? Such an investigation would require more observations in the future. No conclusion can be drawn yet on whether or not A1 and Ax are correlated or anti-correlated until further correlation tests are initiated. This could validate or invalidate Moskalik's claim that they are indeed anti-correlated. Also these findings would need to be compared with other RRc variable stars to truly confirm our results. There may be stronger evidence for (1/2)Fx and Fx being correlated but again more correlation tests are needed.

7 Acknowledgements

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